

Technical Report Documentation Page

1. Report No. Not Assigned PRR-126		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle  Finite Element Models for the 3-Web Sheave Design for the Shippingsport Vertical Lift Bridge				5. Report Date June, 1997	
				6. Performing Organization Code	
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9. Performing Organization Name and Address  Artech Engineering PO Box 2062 Darien, IL 60559				10. Work Unit (TRAIS) HR-25-104-97-1	
				11. Contract or Grant No.	
12. Sponsoring Agency Name and Address Illinois Department of Transportation Bureau of Materials & Physical Research Springfield IL 62704				13. Type of Report and Period Covered Interim Report, May-June 1997	
				14. Sponsoring Agency Code	
15. Supplementary Notes This work was conducted jointly with the Bureau of Bridges & Structures and District 3.					
16. Abstract Finite element models for the Shippingsport Vertical Lift Bridge over the Illinois River at LaSalle were created using a 3-web design for the sheaves. The models used both linear and sinusoidal load distributions along the periphery of the sheave. A model of the cable tray was also created. Using an analysis developed at the University of Illinois, 4/5P downward loading and 1/5P per side was used for loading each semi-circular channel. The models were additionally rotated 20° for maximum loading. Peak tensile stresses generated in the cable tray were at 6 ksi, and the peak tensile web stress was 3.3 ksi. The trunnion shaft of 14", reduced to 12", had a concentrated stress of 15 ksi, taking the radius concentration and cable loading into account.					
17. Key Words sheave; shafting; trunnion; finite element models; stresses; vertical lift bridge; movable bridge; Shippingsport Bridge; Illinois River				18. Distribution Statement Unlimited.	
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 44	22. Price

Analysis of the Shippingsport Vertical Lift Bridge  
Sheave Components - May 1997

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## Introduction

The objective of this analysis is to evaluate the new design of the Shippingsport bridge sheave. Several changes were made to the new design as compared to the existing sheave. The primary change is the addition of a center web under the cable tray that extends to the main hub around the trunnion shaft. Additional revisions include a different cutout geometry on the outer webs. The top of these cutouts were previously shown to have a peak stress area at the top near the minimum height section under the cable tray.

The primary loading has been determined to be due to the counter balance weight that hangs from two of these sheaves by way of two pairs of eight cables. The total weight of the counter balance has been revised from 500 kips to 443 kips for this new analysis. A drawing of the counter balance and sheave overall loading is shown on page A.1 in the appendix.

The pressure distribution that is created on the sheave by the cables is the main loading that was considered in this analysis. For the purpose of this study, two loading distributions were used; linear and sinusoidal. The linear distribution is based on a constant normal pressure over the portion of the sheave that is in contact with the cables. The sinusoidal distribution is based on a maximum normal pressure at the top of the sheave that reduces to zero as the cables leave the tray. The reduction of the normal pressure from a maximum to zero decays by way of a sine function which is why it is called a sinusoidal distribution. Both of these load derivations are included in the body of the report.

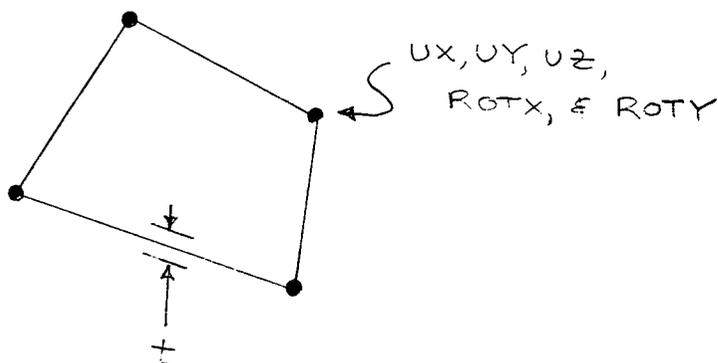
In addition to using two different load distributions, two sheave orientations were also investigated. Since the sheave rotates during the raising of the counter balance weight, these loads will act on the sheave model at various locations. The two orientations that were analyzed are zero offset and a 20 degree rotational offset.

A cross section of the cable tray is also analyzed in this report. The loading for this model is based on a load derivation that considers the geometry of the tray and the make-up of the each the eight cables. Finally, The trunnion shaft was also analyzed and is presented at the end of this report.

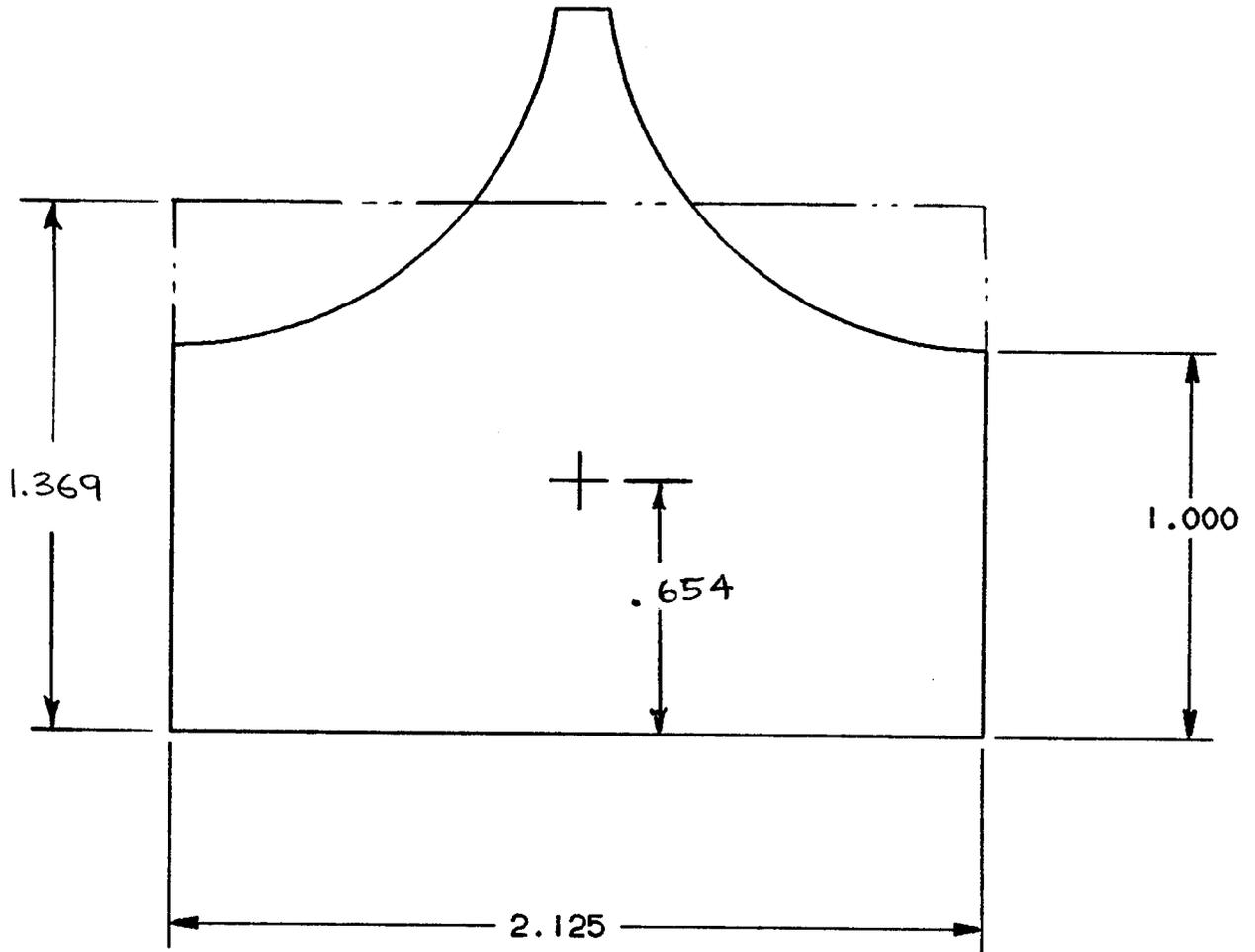
The FEA (Finite Element Analysis) models that were developed in this analysis for the sheave, were made using 3-D plate elements. These elements have 5 degrees of freedom; translations in X, Y, and Z and two rotations. These elements are initialized using an input parameter for the thickness. For the sheave geometry, the thickness was set for the various plate sizes used in the design. The element type and thickness setting give membrane and bending stress output so that the stress on the surface of the various plates can be viewed.

The following properties for the steel materials were used in this analysis:

Modulus of Elasticity	(psi)	30,000,000
Poisson's Ratio		.3



SOLVE FOR EFFECTIVE RIM THICKNESS  
FOR FEA MODEL



AREA = 2.66290  
 XBAR = -1.95000  
 YBAR = 2.99693  
 I Y-Y = .849938  
 I X-Y = -.000000  
 I X-X = .454817  
 R Y-Y = .564958  
 R X-X = .413277  
 ZETA = -.000043  
 IP Y-Y = .849938  
 IP X-X = .454817  
 RP Y-Y = .564958  
 RP X-X = .413277

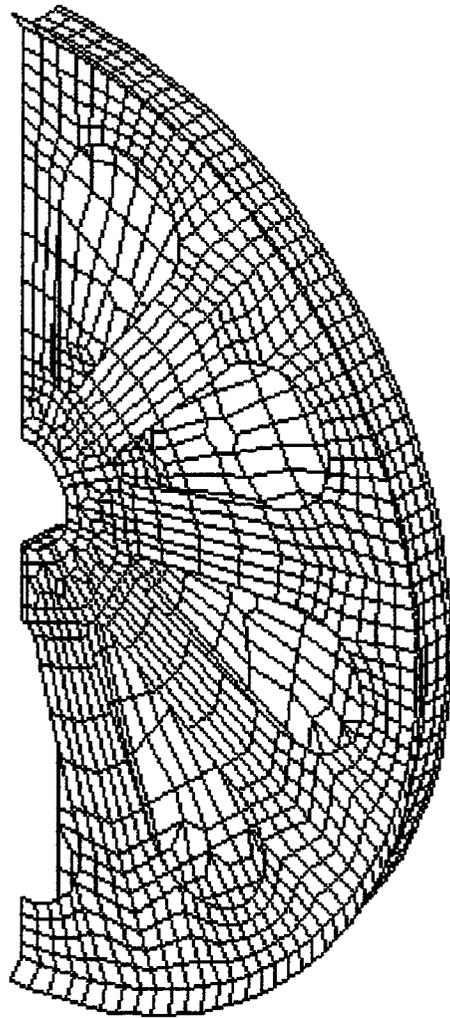
$$I_{x-x} = \frac{bh^3}{12}$$

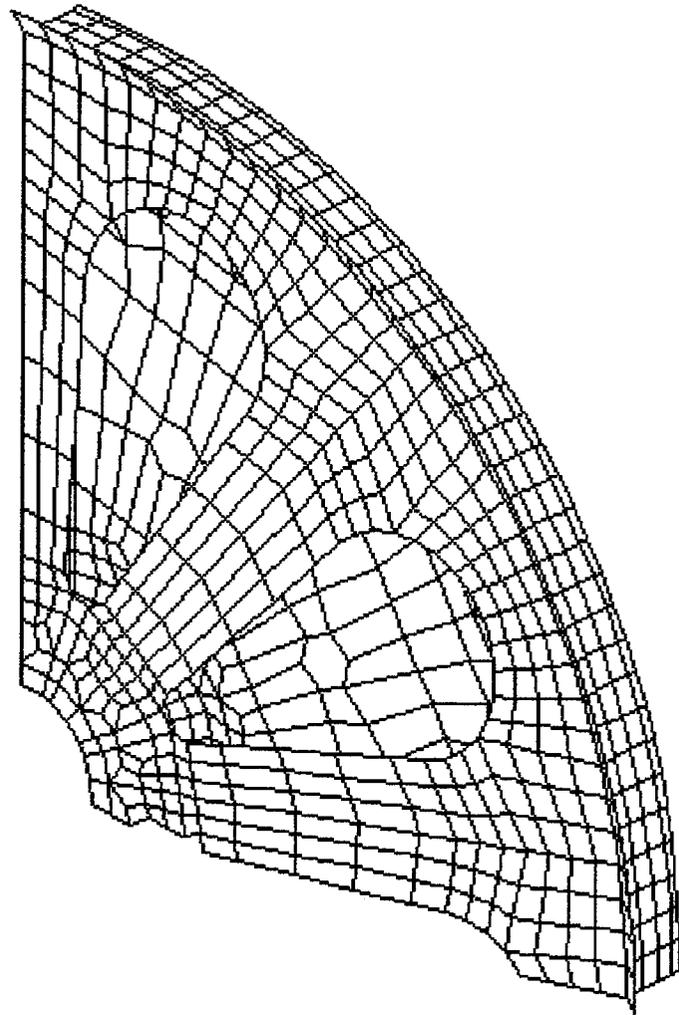
$$h = \sqrt[3]{\frac{.454817 (12.0)}{2.125}}$$

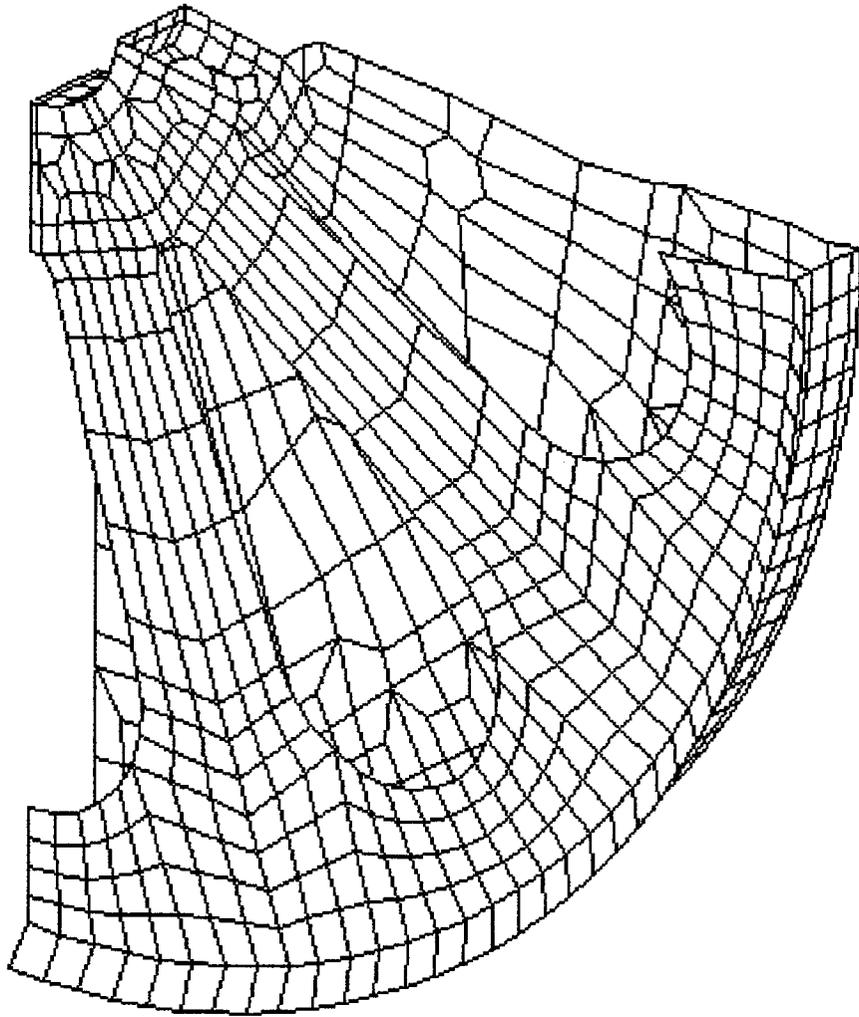
$$h = 1.369 \text{ IN}$$

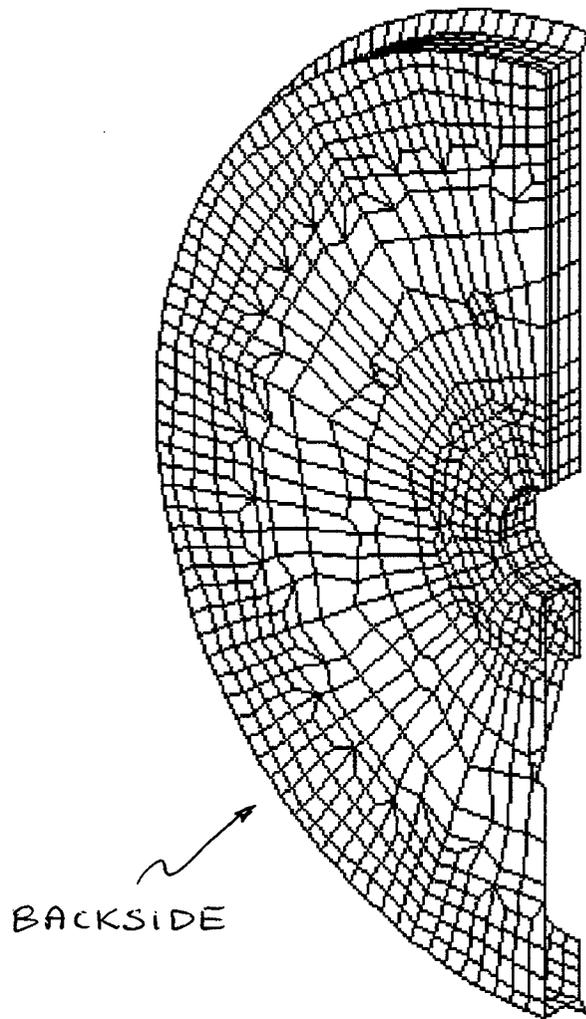
TIDT SECTI , AJR  
DOUBLE SCALE

FEA 1/4 MODEL OF THE SHEAVE - NO OFFSET

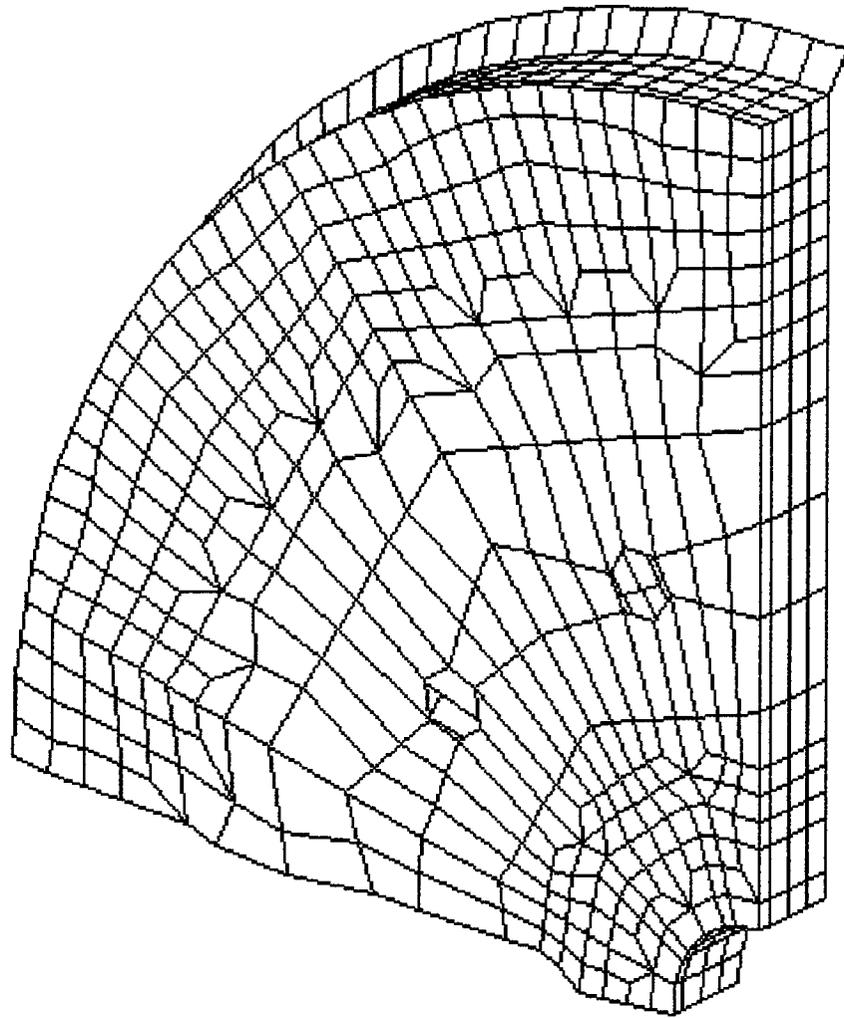


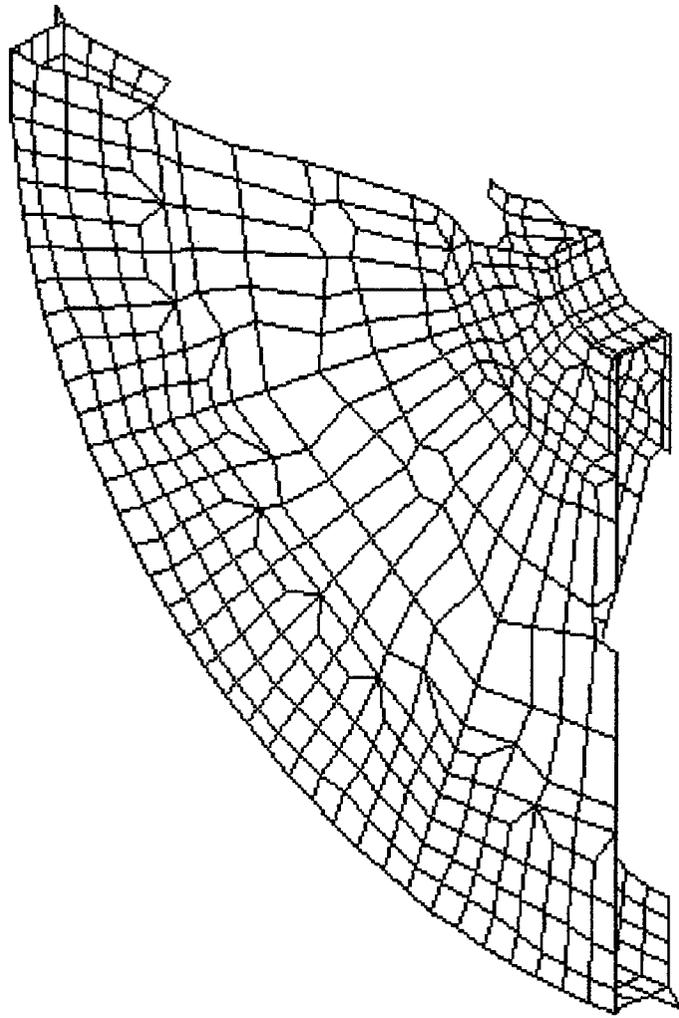






BACKSIDE





# LINEAR LOAD DISTRIBUTION

$$P = \frac{2T}{D}$$

FOR THE FEA 1/4 MODEL,

$$T = \frac{221,653 \text{ LB}}{2} = 110,826.5 \text{ LB}$$

$$P = \frac{2(110,826.5)}{150} = 1478 \text{ LB/IN}$$

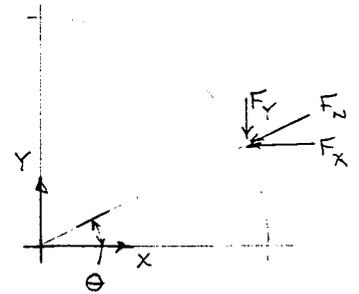
$$F_N = P \times l$$

FOR 10° INCREMENTS,

$$l = r \times \theta$$

$$l = 75 \text{ IN} (.1745 \text{ RAD}) = 13.09 \text{ IN}$$

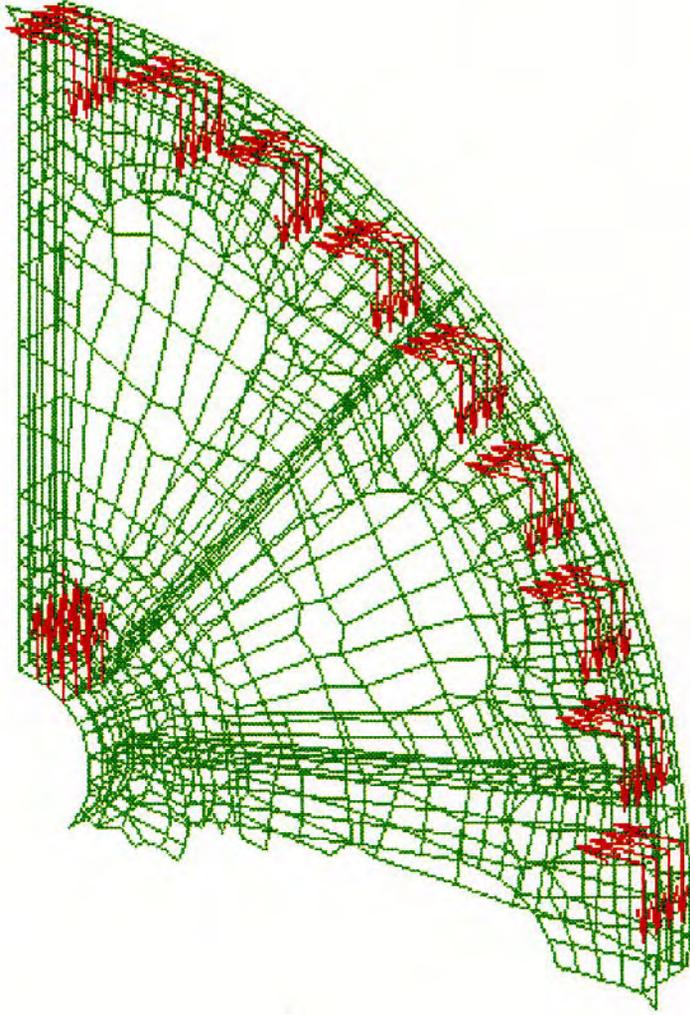
$$F_N = 1478 \frac{\text{LB}}{\text{IN}} (13.09 \text{ IN}) = 19,347 \text{ LB}$$



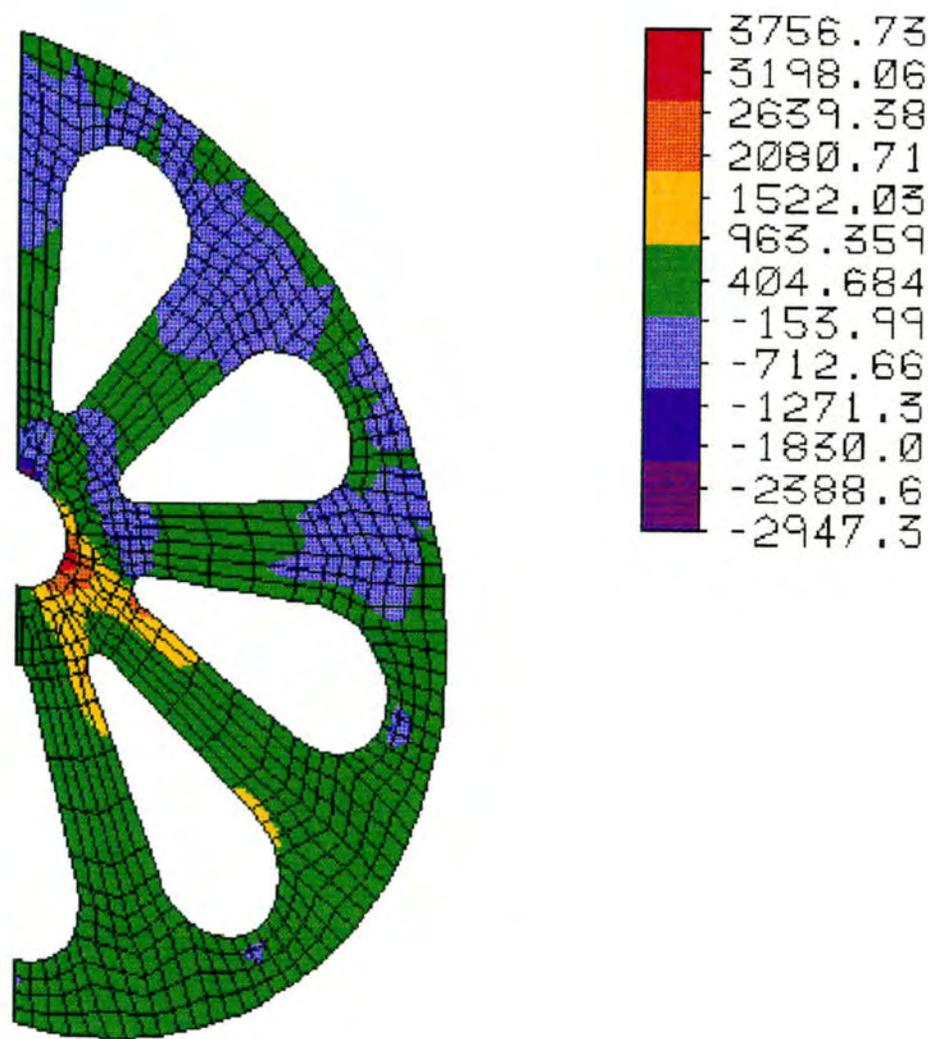
#	θ LOCATION FOR LOAD	P	F <sub>N</sub>	F <sub>x</sub>	F <sub>y</sub>
1	85°	1478 $\frac{\text{LB}}{\text{IN}}$	19,347 LB	1686 LB	19,273 LB
2	75°			5007	18,688
3	65°			8176	17,534
4	55°			11,097	15,848
5	45°			13,680	13,680
6	35°			15,848	11,097
7	25°			17,534	8176
8	15°			18,688	5007
9	5°			19,273	1686

110,989 LB ✓

LOAD CASE 1

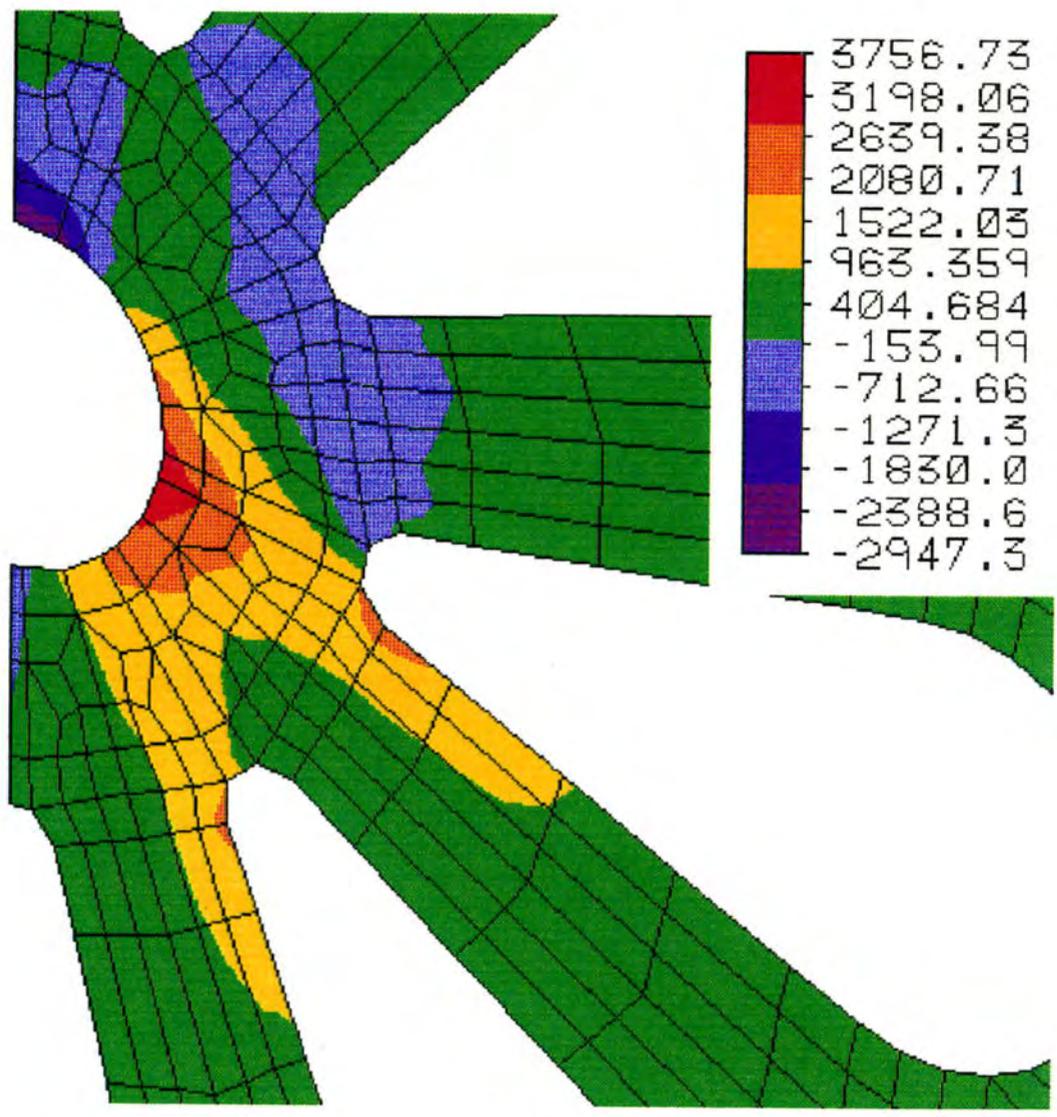


LOADING PLOT



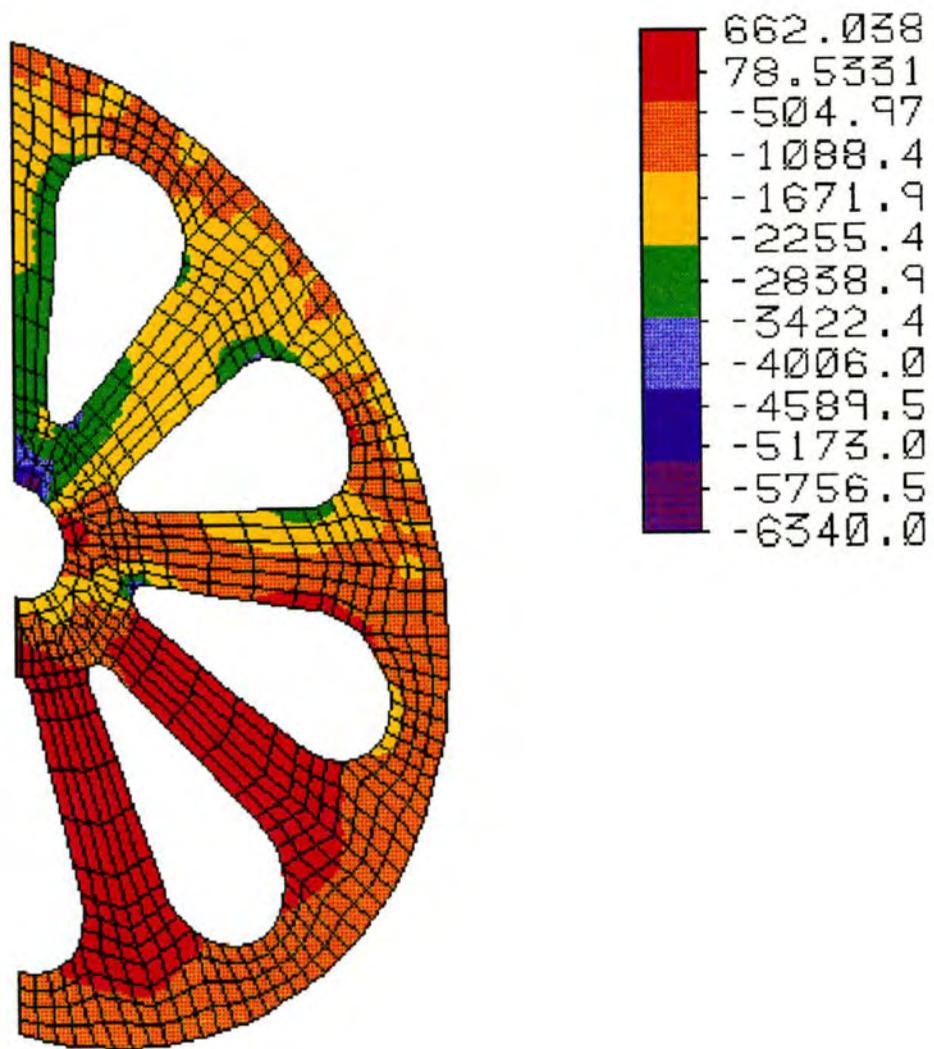
FRONT WEB

MAXIMUM PRINCIPLE TENSILE STRESS (psi)

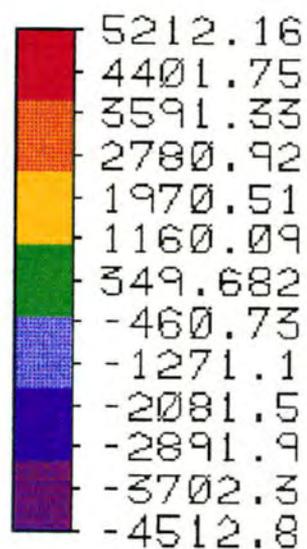
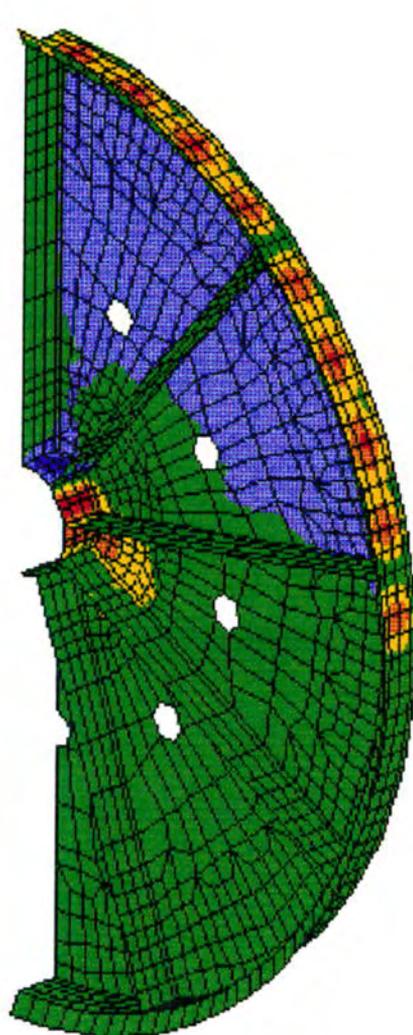


CLOSE-UP OF HIGH STRESS AREA

MAXIMUM PRINCIPLE TENSILE STRESS (psi)

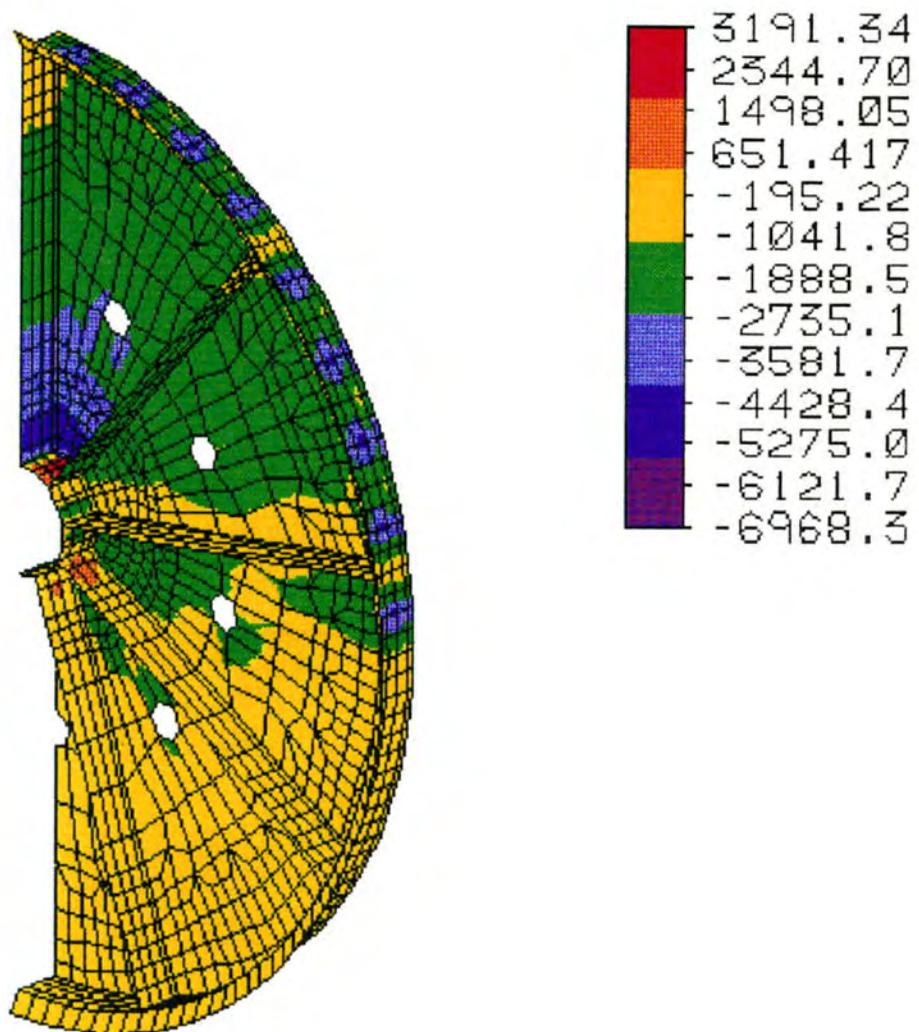


MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)



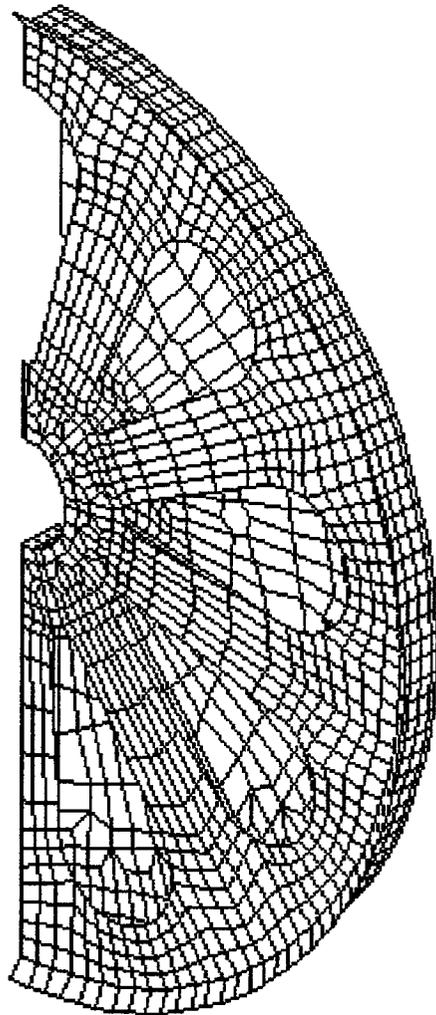
FRONT WEB REMOVED

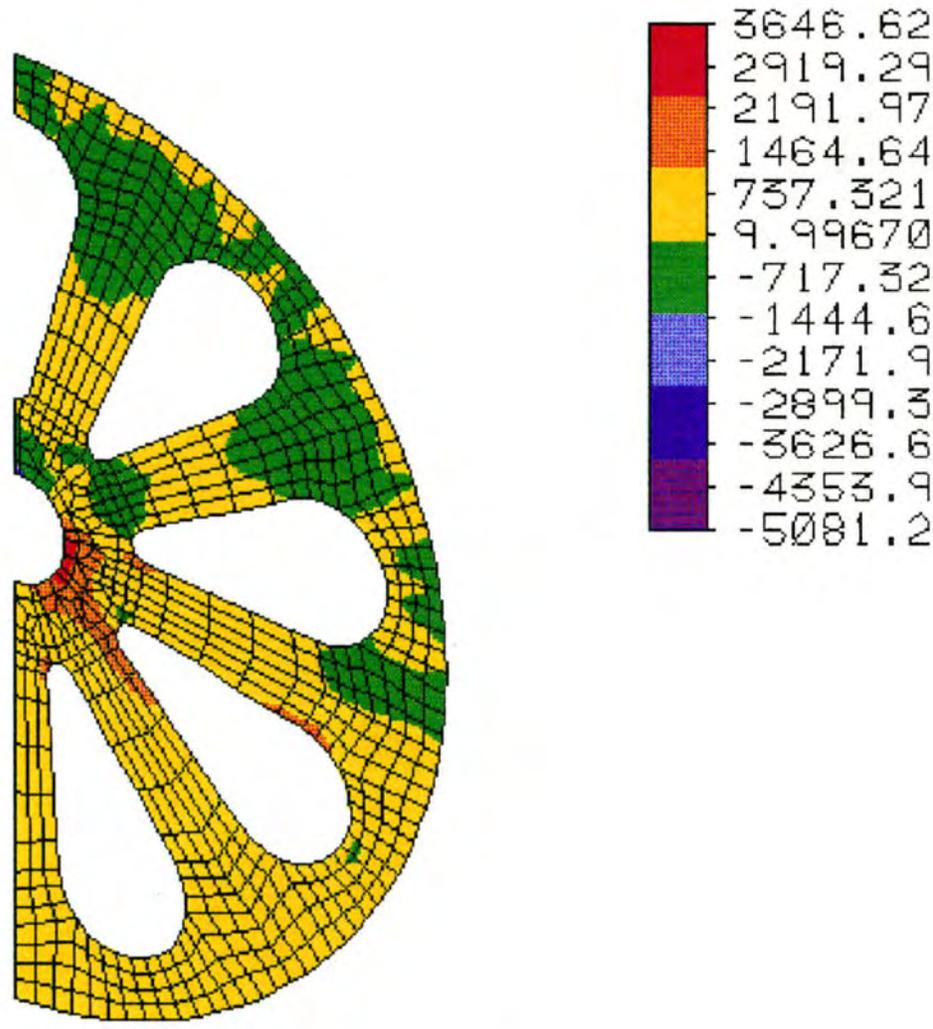
MAXIMUM PRINCIPLE TENSILE STRESS (psi)



MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)

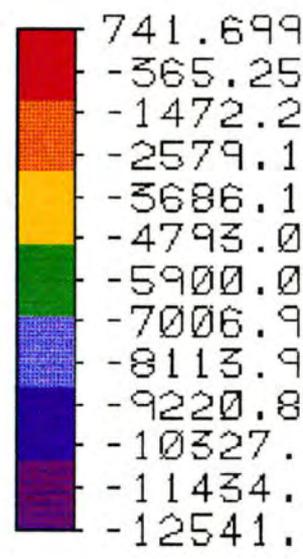
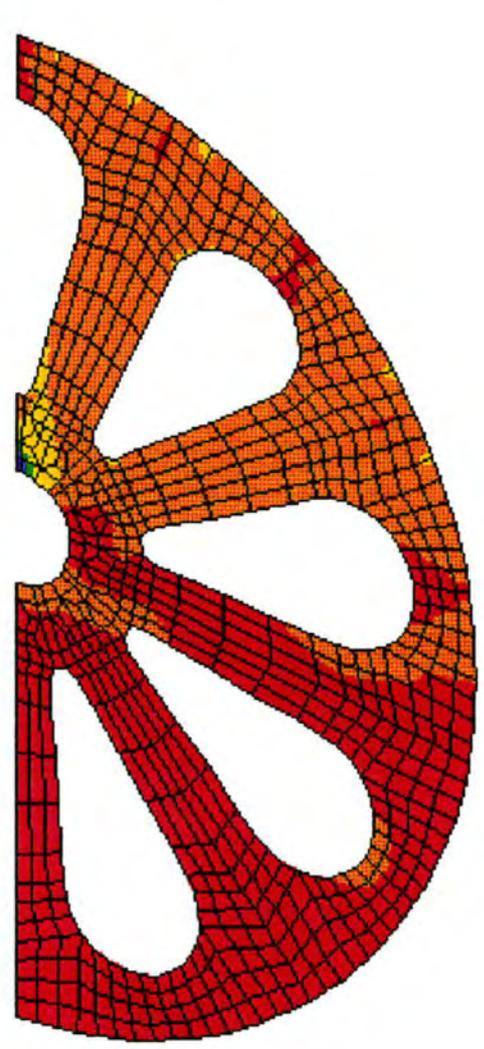
LOAD CASE 2 - 20° ROTATIONAL OFFSET



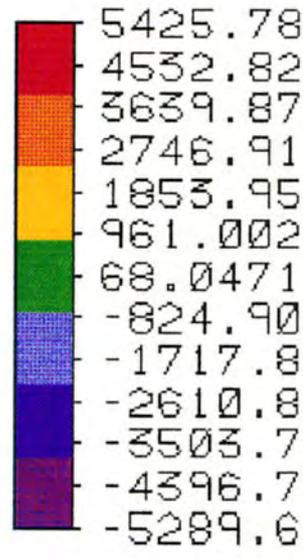
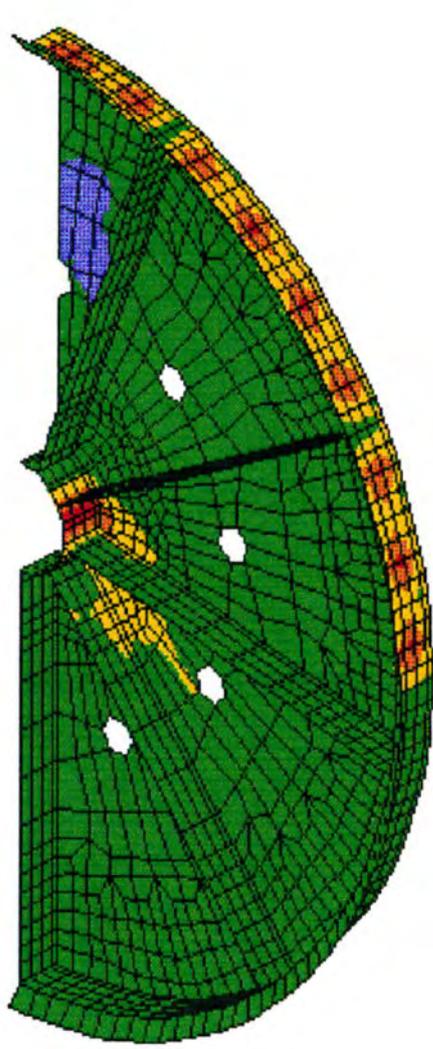


FRONT WEB

MAXIMUM PRINCIPLE TENSILE STRESS (psi)

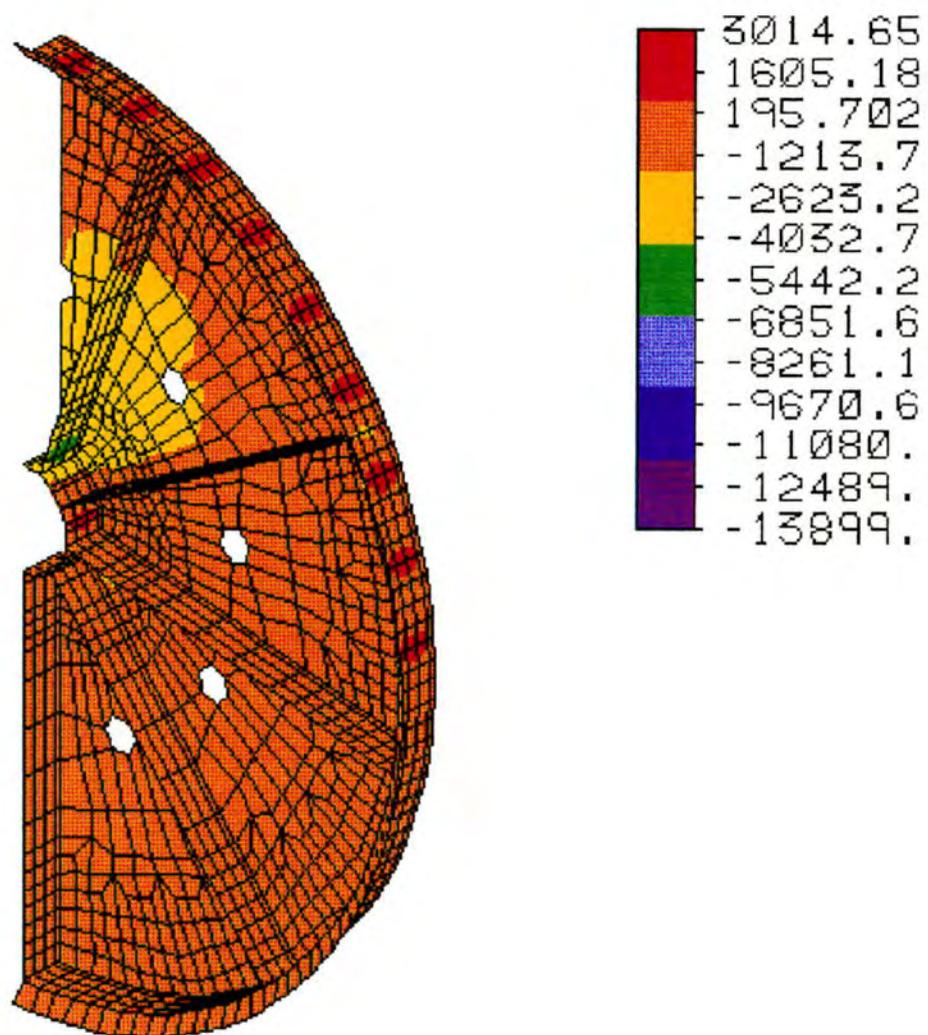


MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)



FRONT WEB REMOVED

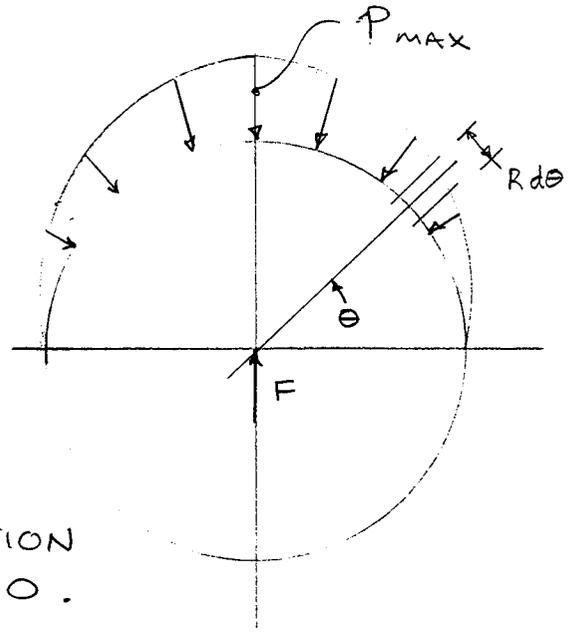
MAXIMUM PRINCIPLE TENSILE STRESS (psi)



MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)

## SINUSOIDAL LOAD DISTRIBUTION

THE EFFECTIVE NORMAL LOAD ON THE SHEAVE SHOULD APPROACH ZERO AT THE POINT THE ROPES COME OUT OF THE TRAY.



LET THE  $P$  DISTRIBUTION GO FROM  $P_{max}$  TO  $0$  USING A SINE FUNCTION OVER  $\theta = \frac{\pi}{2}$  TO  $\theta = 0$ .

NOW,

$$P = P_{max} (\sin \theta) \quad \textcircled{1}$$

$$dF = P \times R d\theta \times \sin \theta \quad \textcircled{2}$$

PUT  $\textcircled{1}$  IN  $\textcircled{2}$

$$dF = [P_{max} (\sin \theta)] R d\theta \times \sin \theta$$

$$dF = P_{max} R \sin^2 \theta d\theta$$

$$\frac{F}{2} = P_{max} R \int_0^{\frac{\pi}{2}} \sin^2 \theta d\theta$$

NOTE  $\frac{F}{2} = T$ , WHICH IS THE TENSION IN THE CABLE

$$T = P_{\max} R \left[ \frac{1}{2} \theta - \frac{1}{4} \sin(2\theta) \right]_0^{\frac{\pi}{2}}$$

$$T = P_{\max} R \left[ \frac{\pi}{4} - (0 - 0) \right]$$

$$T = \frac{\pi}{4} P_{\max} R$$

$$P_{\max} = \frac{4T}{\pi R}$$

FOR THE FEA 1/4 MODEL,

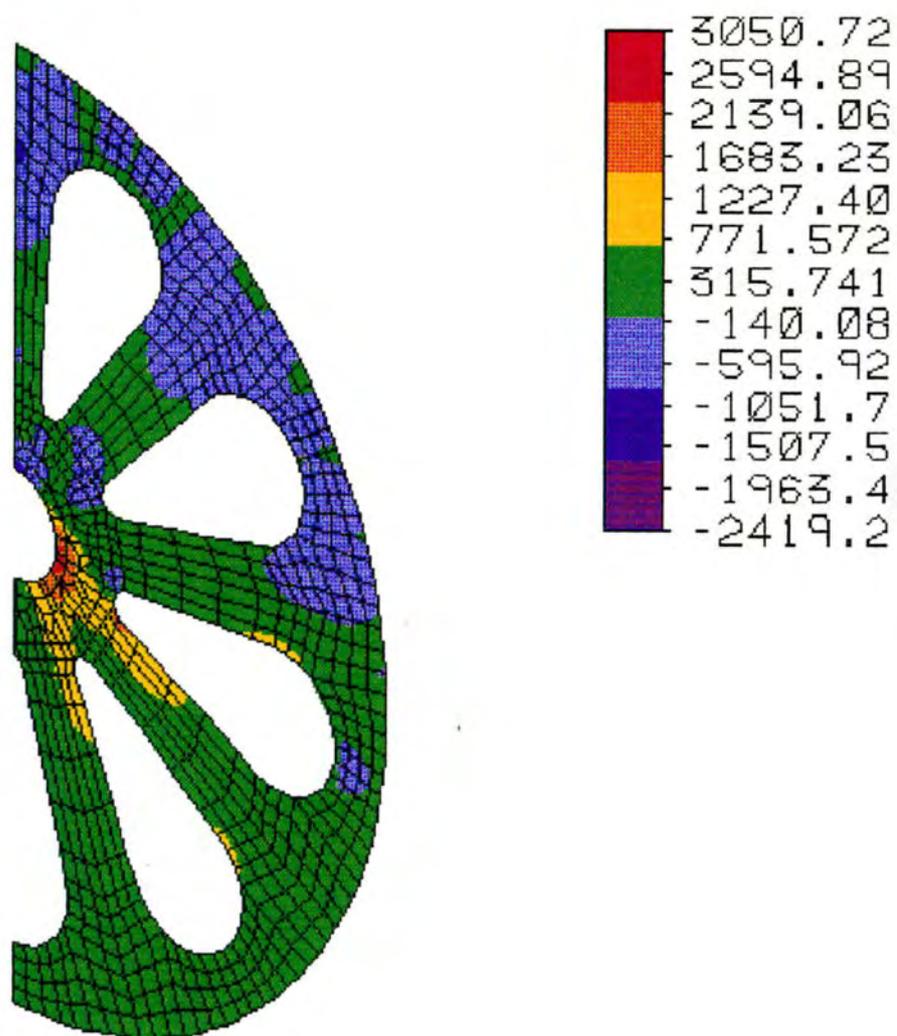
$$T = 110,826.5 \text{ LB}$$

NOW,

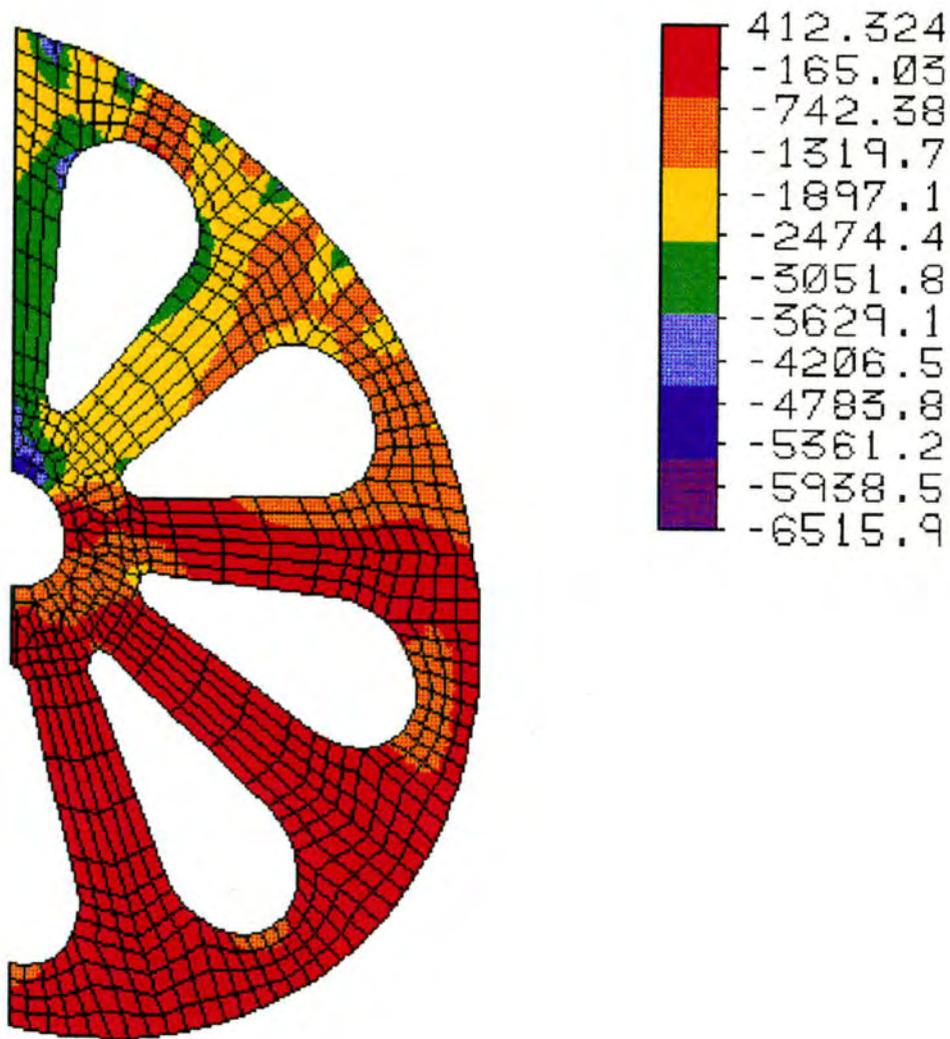
$$P_{\max} = \frac{4(110,826.5 \text{ LB})}{\pi(75 \text{ IN})} = 1881 \frac{\text{LB}}{\text{IN}}$$

#	θ LOCATION FOR LOAD	P	F <sub>N</sub>	F <sub>x</sub>	F <sub>y</sub>
1	85°	1874 $\frac{\text{LB}}{\text{IN}}$	24,531 LB	2138 LB	24,438 LB
2	75°	1817	23,785	6156	22,975
3	65°	1705	22,318	9432	20,227
4	55°	1541	20,172	11,570	16,524
5	45°	1330	17,410	12,311	12,311
6	35°	1079	14,124	11,570	8101
7	25°	795	10,407	9432	4398
8	15°	487	6375	6158	1650
9	5°	164	2147	2139	187

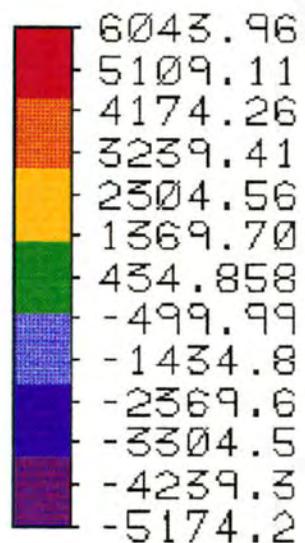
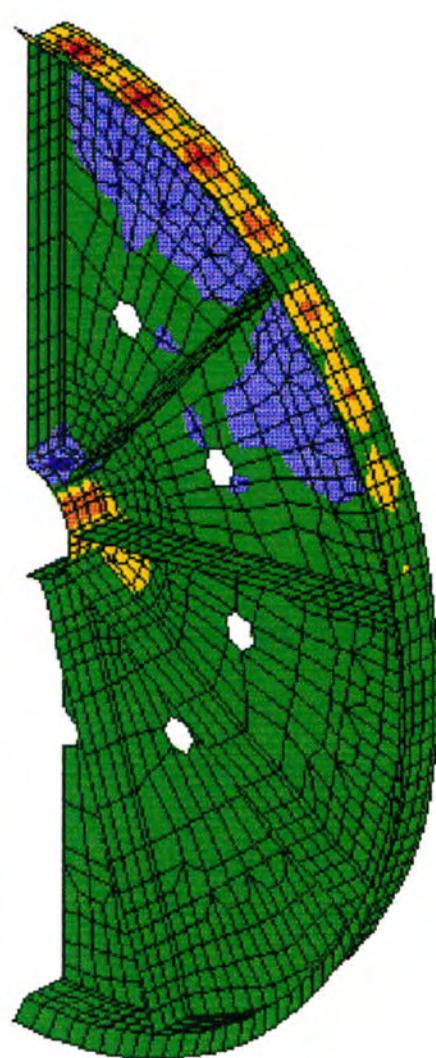
110,811 LB ✓



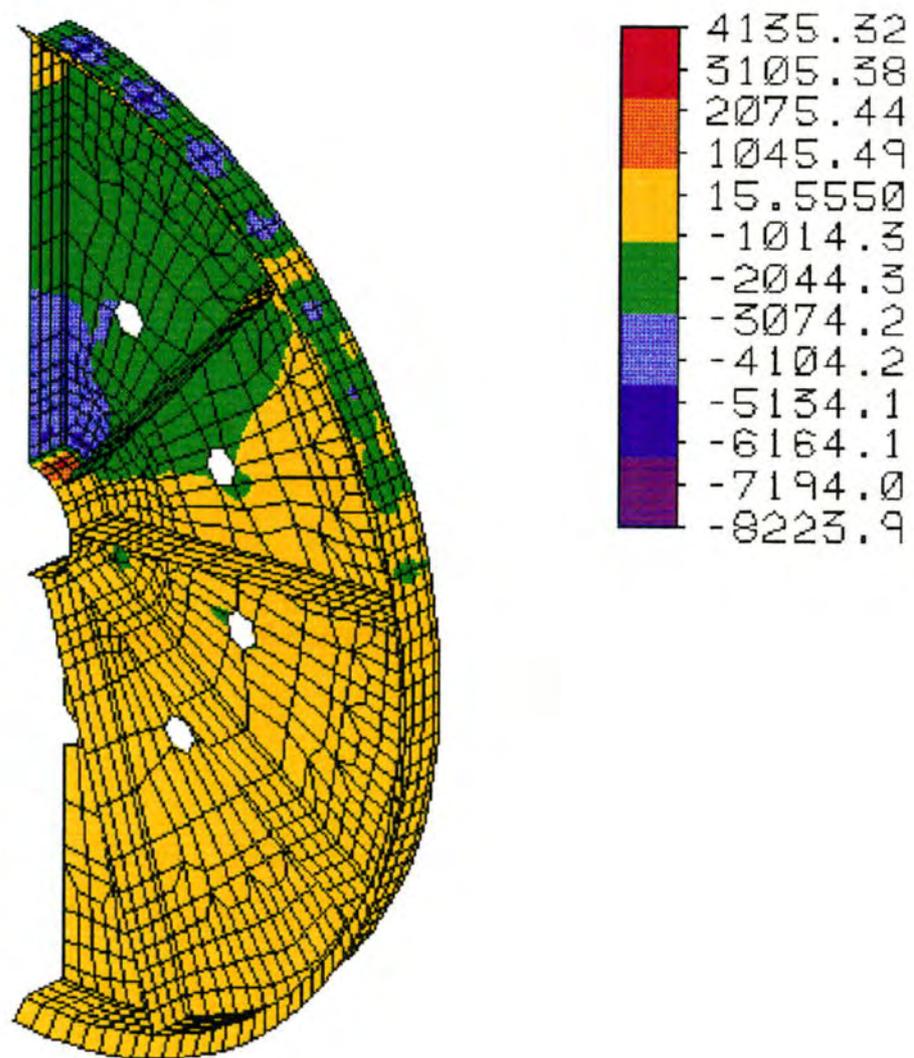
MAXIMUM PRINCIPLE TENSILE STRESS (psi)



MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)

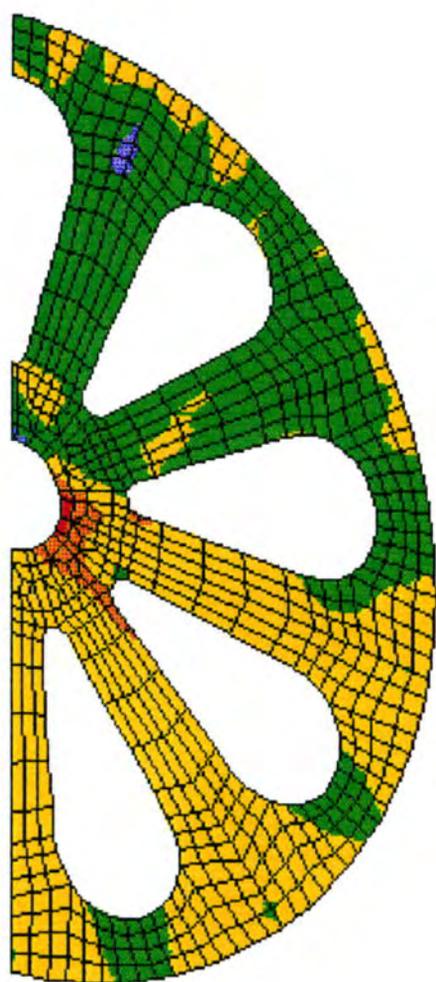


MAXIMUM PRINCIPLE TENSILE STRESS (psi)



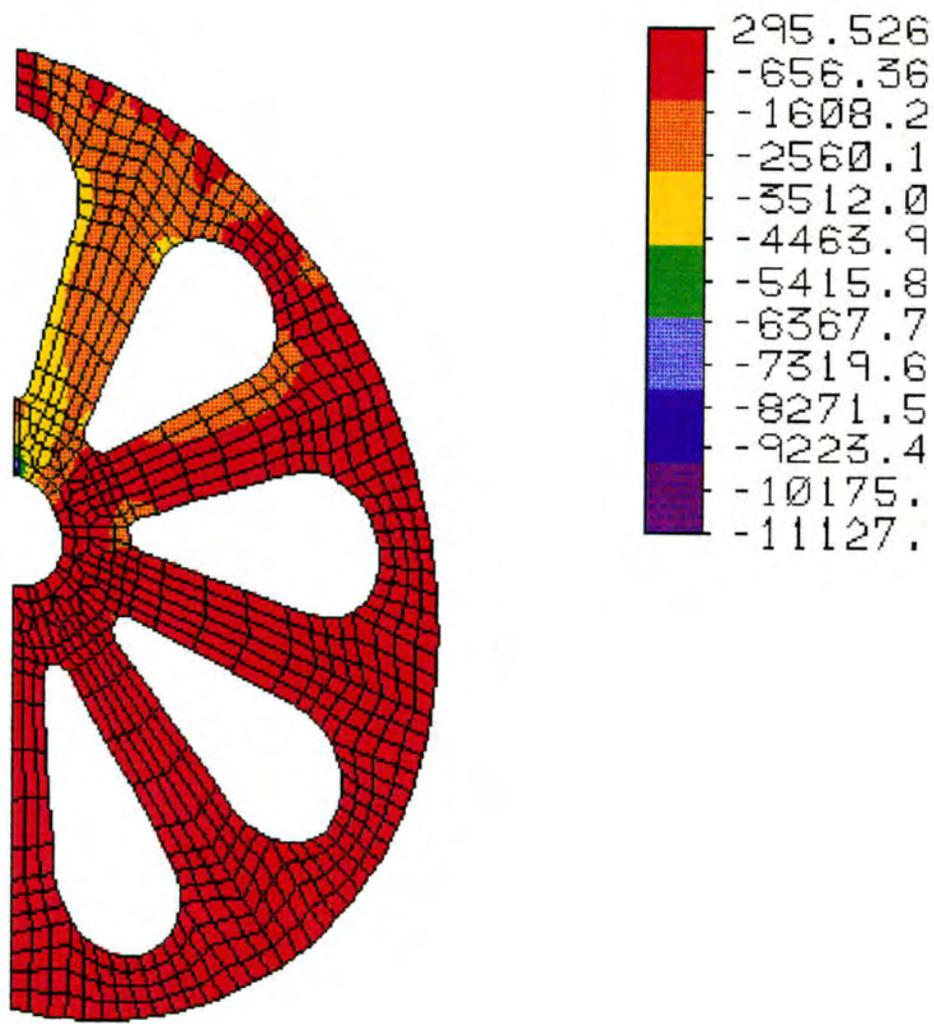
MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)

LC 4 - 20° ROTATIONAL OFFSET

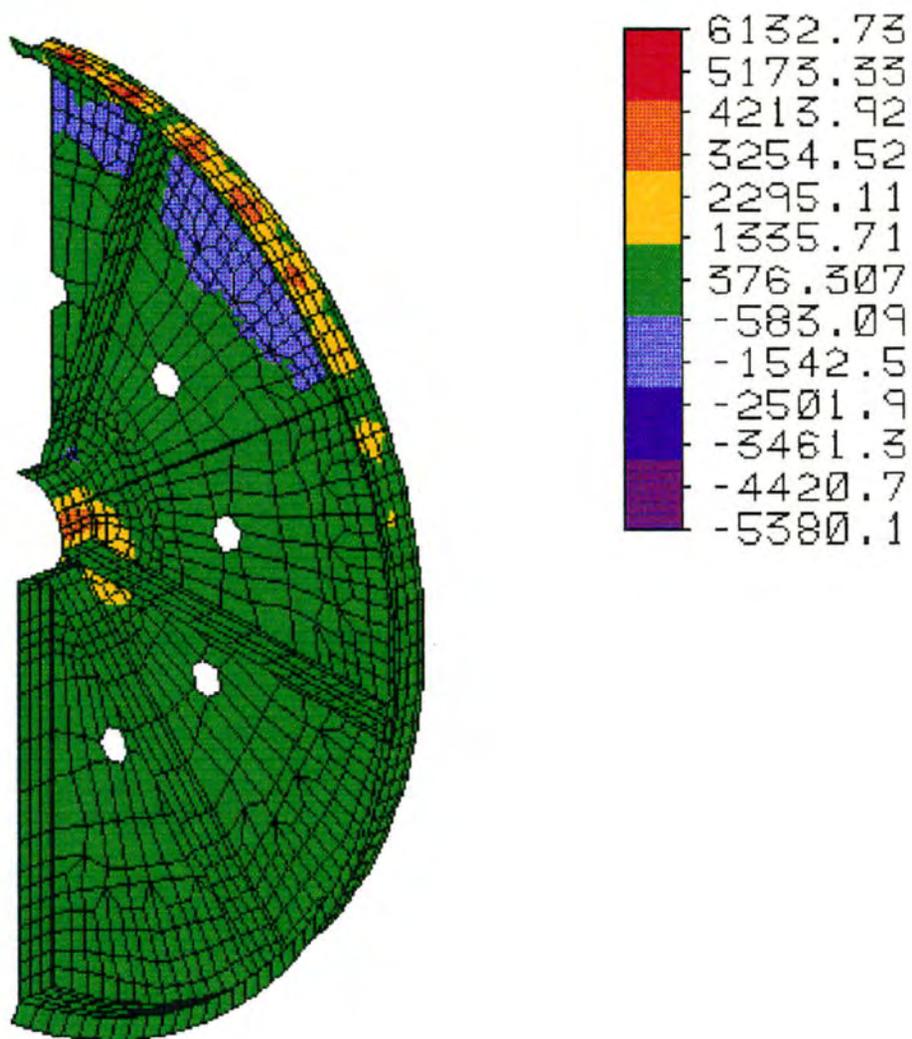


FRONT WEB

MAXIMUM PRINCIPLE TENSILE STRESS (psi)

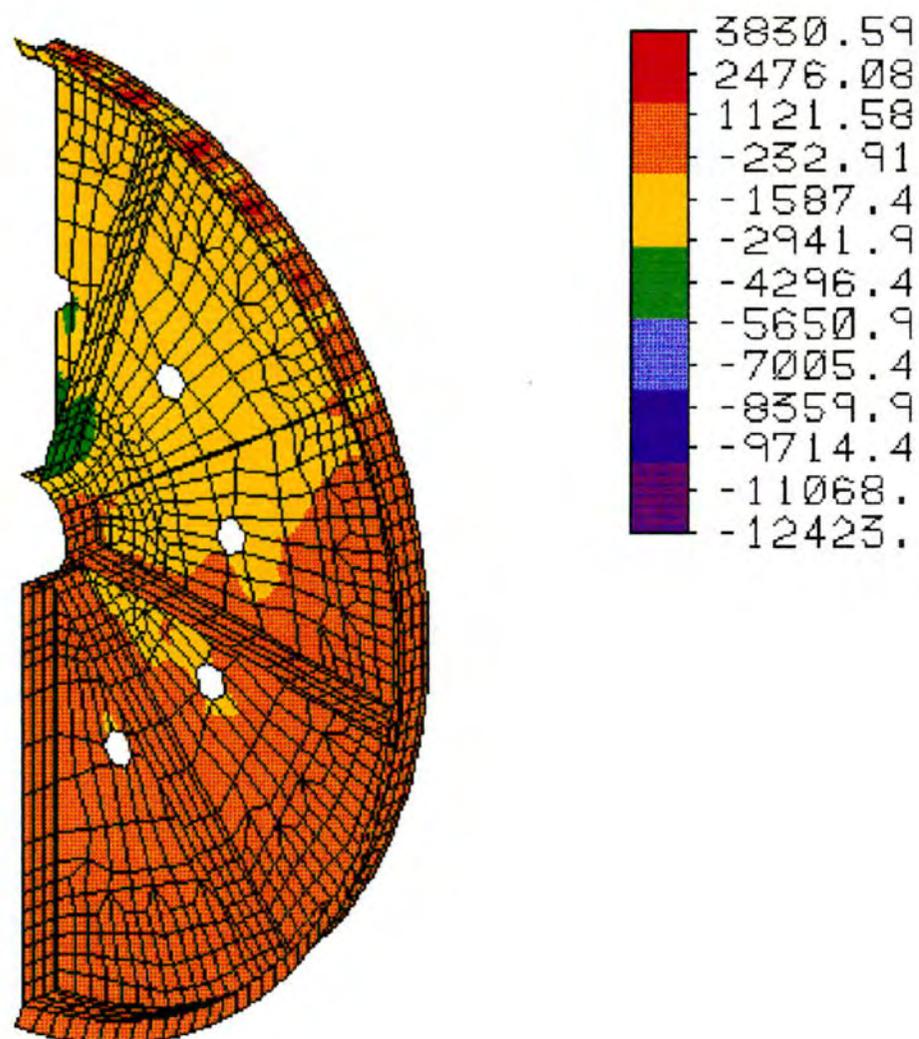


MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)



FRONT WEB REMOVED

MAXIMUM PRINCIPLE TENSILE STRESS (psi)



MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)

## CABLE TRAY LOADING

USING THE LINEAR LOAD DISTRIBUTION, THE NORMAL LOADING FROM THE CABLE IS ;

$$P = \frac{1478 \text{ LB/IN}}{4} = 369.5 \text{ LB/IN} \quad (\text{PER CABLE})$$

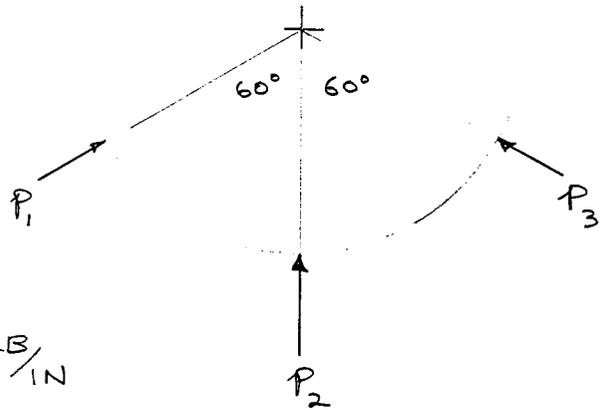
4 CABLES FOR THE 1/4 MODEL

FROM THE CABLE TRAY LOADING DERIVATION,

$$P_1 = P_3 = \frac{1}{5} P$$

$$P_1 = P_3 = \frac{369.5}{5} = 73.9 \text{ LB/IN}$$

$$P_2 = \frac{4}{5} (369.5) = 295.6 \text{ LB/IN}$$



CHECK,

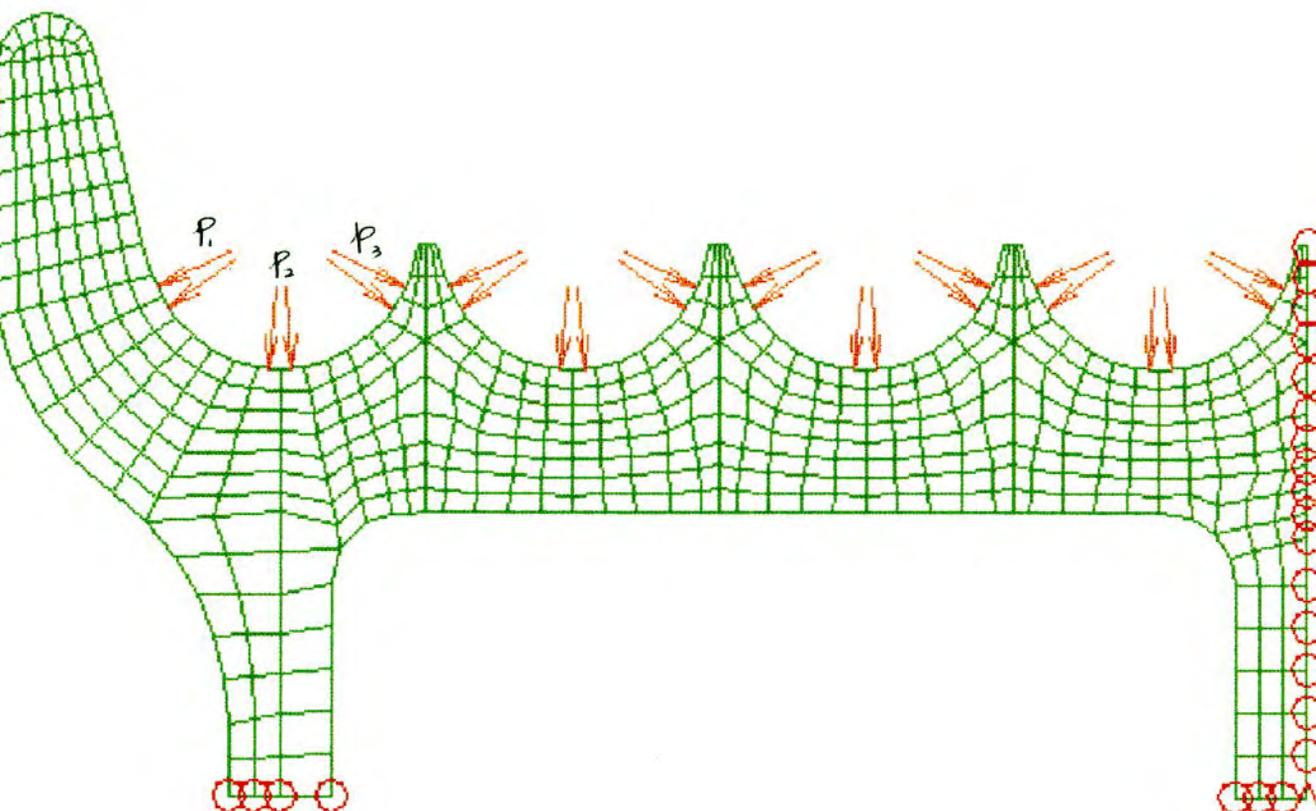
$$[73.9 (\sin 30^\circ)] 2 + 295.6 = 369.5 \text{ LB/IN} \quad \checkmark$$

FOR MODEL,

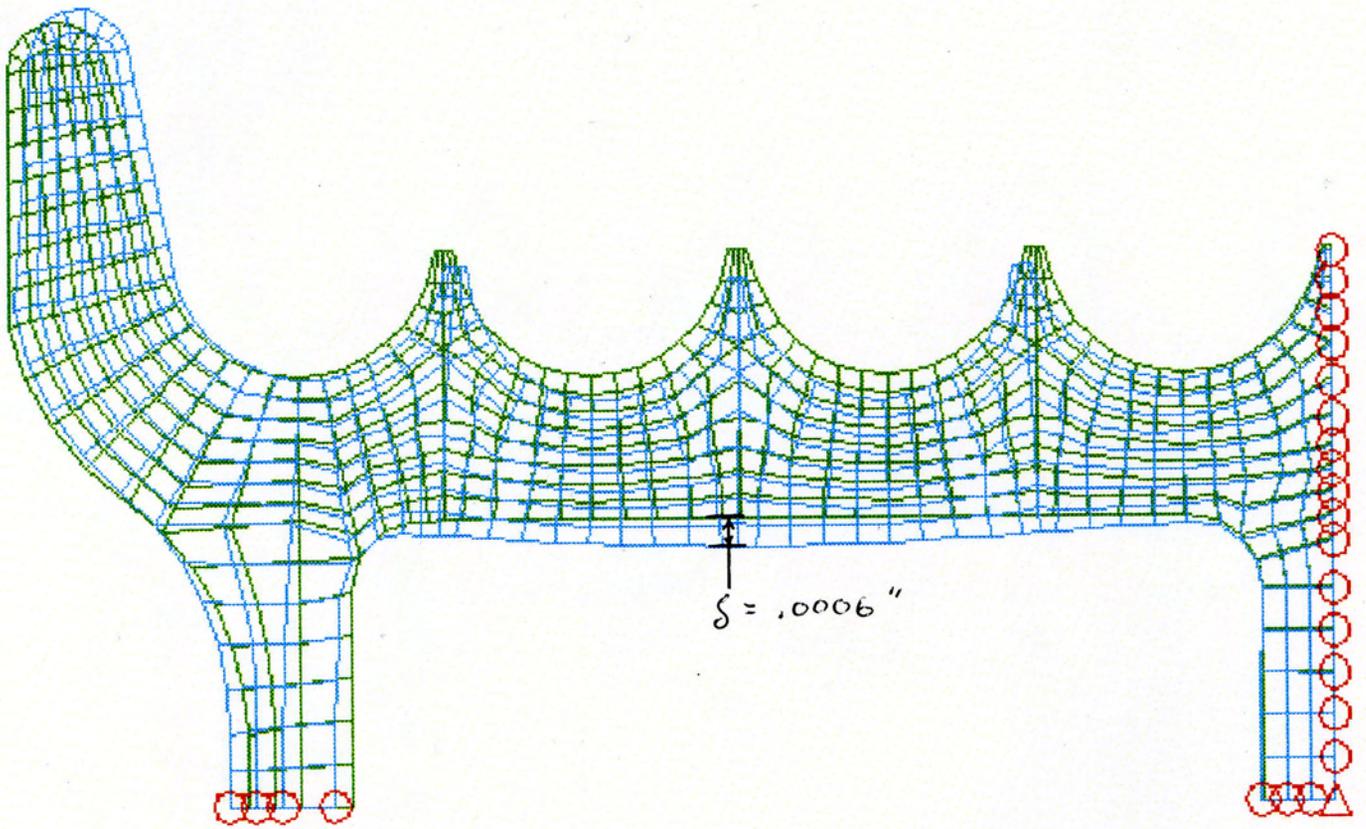
$$q_1 = q_3 = \frac{73.9}{.3486} = 212 \text{ psi}$$

$$q_2 = \frac{295.6}{.3486} = 848 \text{ psi}$$

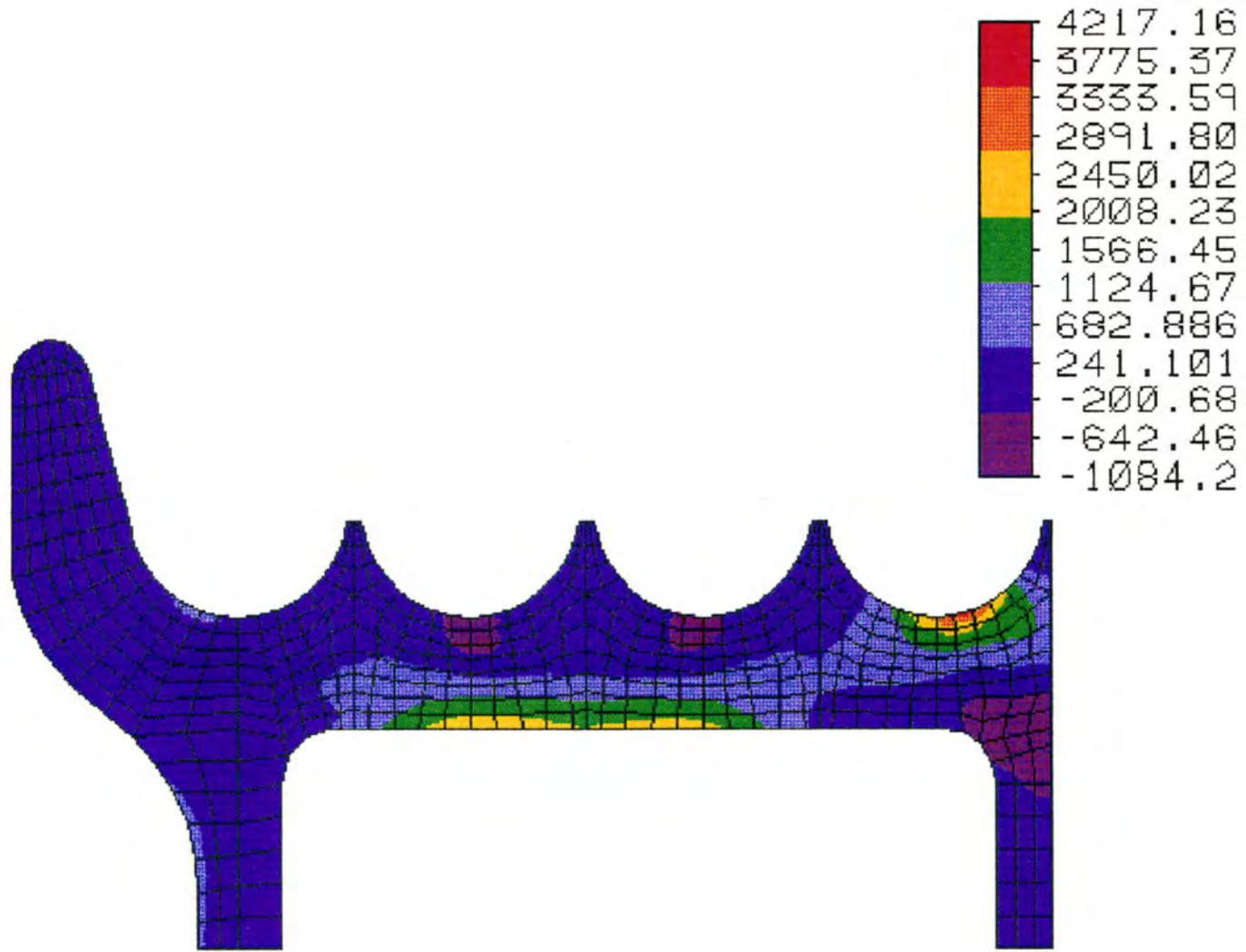
CABLE TRAY CROSS SECTION



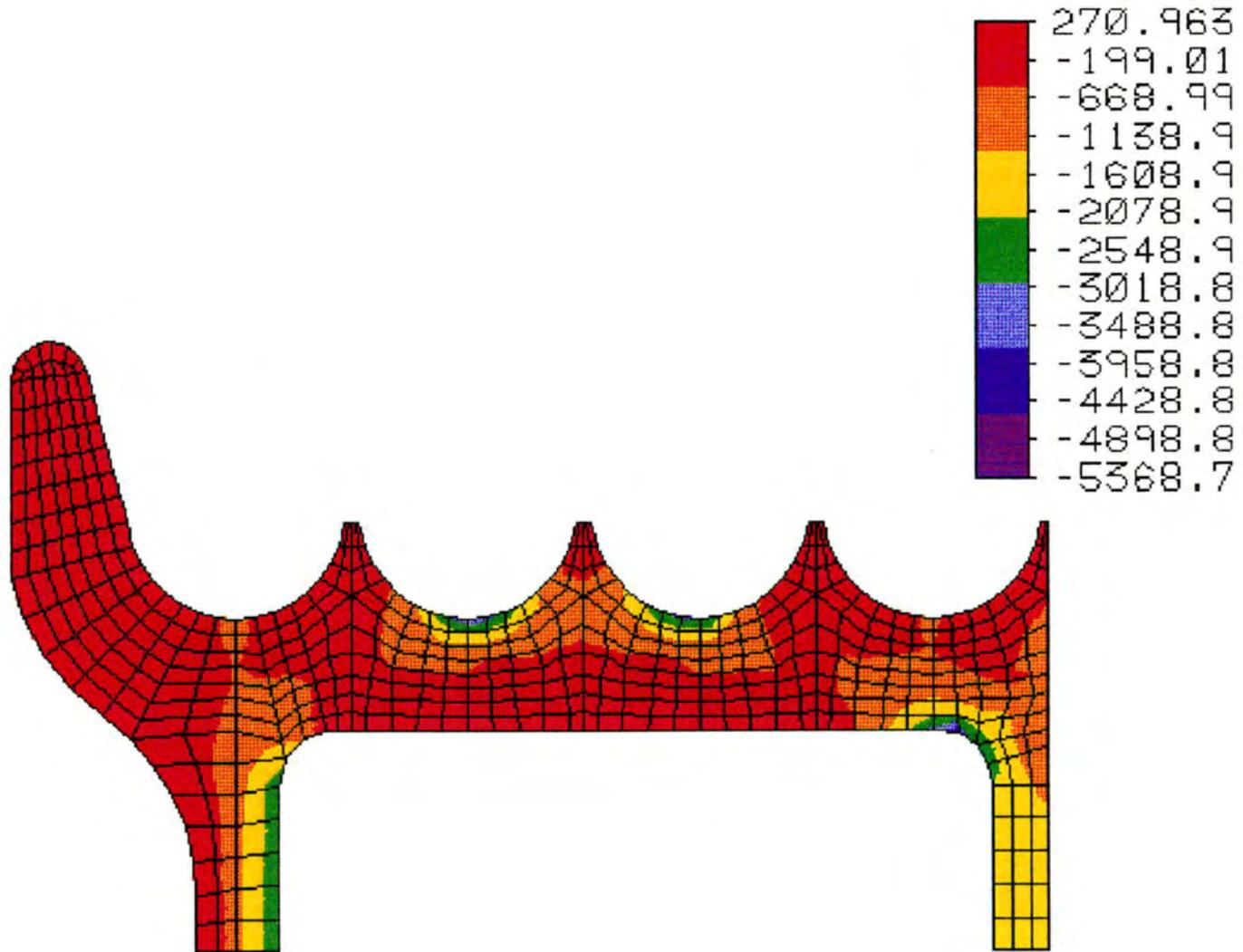
LOADING AND BOUNDARY CONSTRAINTS



DEFLECTED SHAPE SUPER-IMPOSED ON THE MODEL



MAXIMUM PRINCIPLE TENSILE STRESS (psi)



MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)

# TRUNNION SHAFT ANALYSIS

BEARING PRESSURE  
BETWEEN SHEAVE  
AND SHAFT IS

$$\sigma_c = \frac{F}{DIA \times L}$$

$$\sigma_c = \frac{443,306 \text{ LB}}{14.0 \text{ IN} (20.0 \text{ IN})}$$

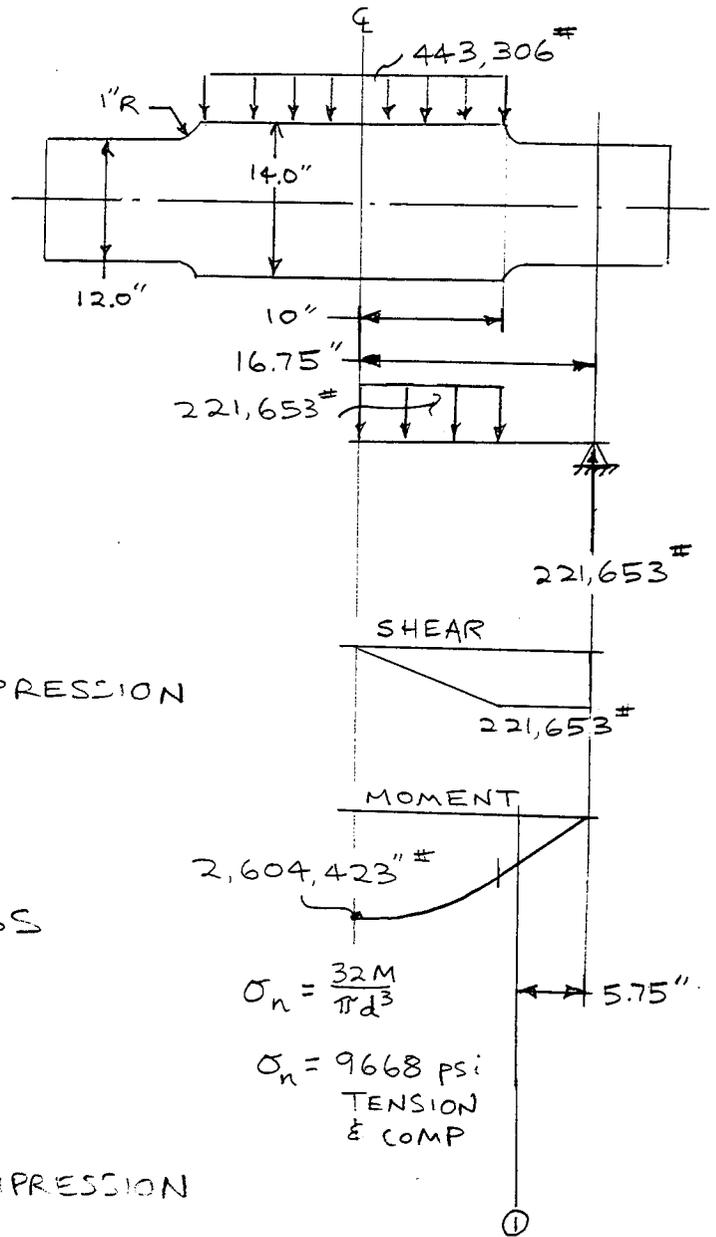
$$\sigma_c = 1583 \text{ psi COMPRESSION}$$

BEARING PRESSURE  
IN BRONZE BUSHINGS

$$\sigma_c = \frac{221,653}{12 (13.0)}$$

$$\sigma_c = 1421 \text{ psi COMPRESSION}$$

BENDING STRESSES FULLY  
REVERSE FROM TENSION  
TO COMPRESSION, BUT  
THE TOTAL CYCLES  
SHOULD BE LOW.



$$M = 1,274,505 \text{ inch-lbs}$$

$$\sigma_n = 7513 \text{ psi TENSION \& COMP.}$$

$$K = 1.65$$

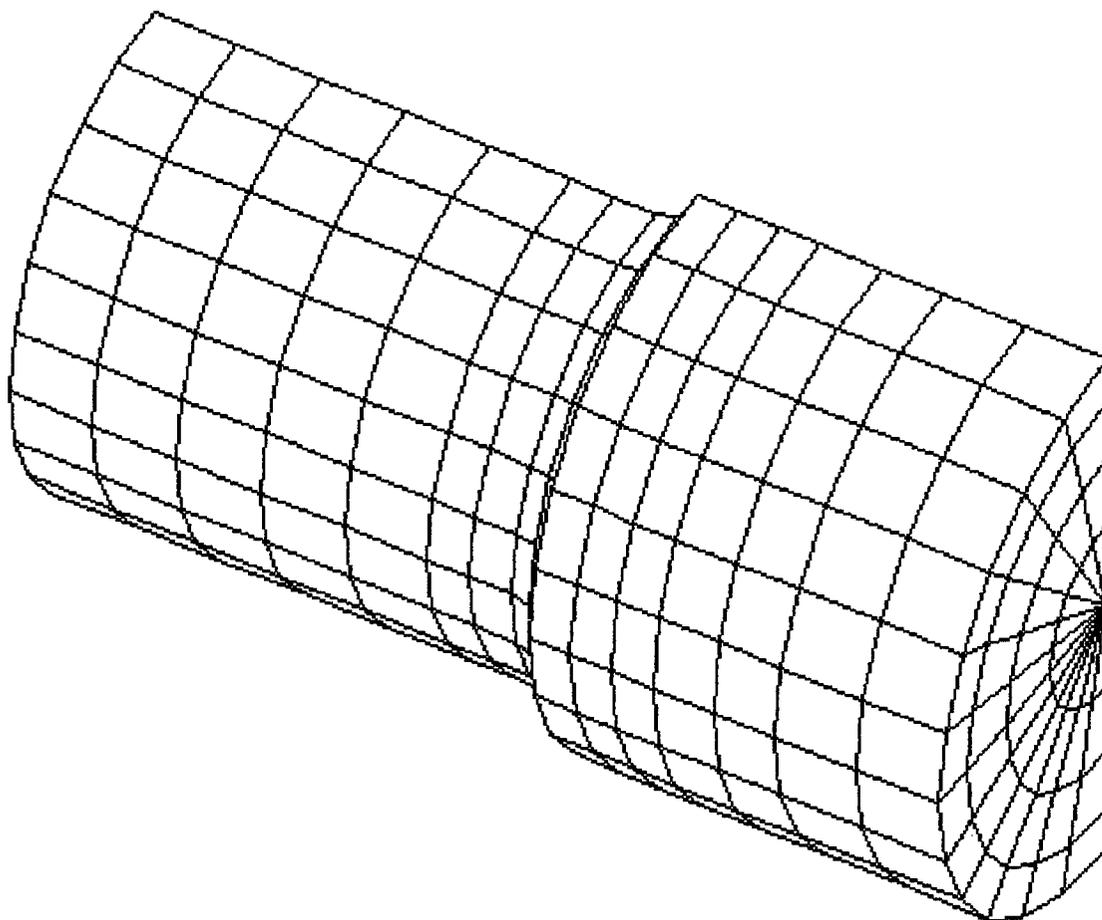
$$\sigma_{max} = K \times \sigma_n$$

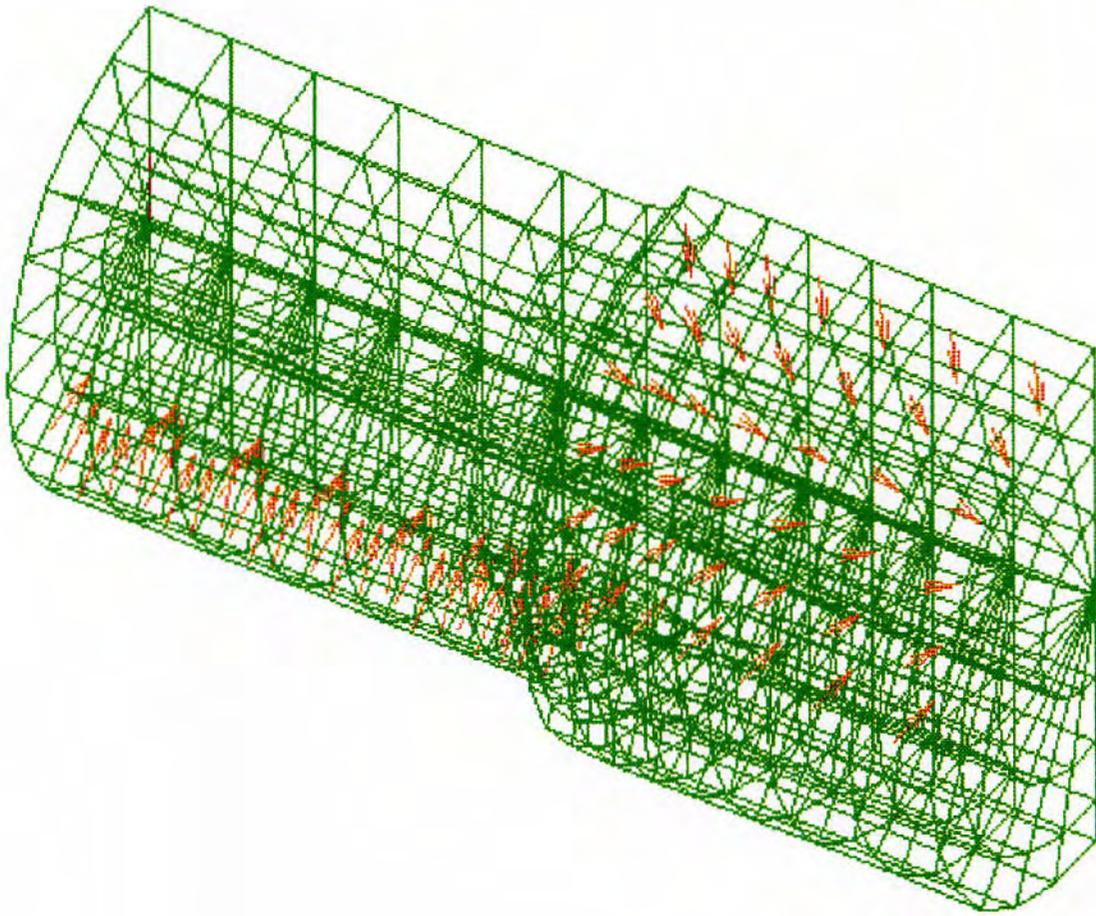
$$\sigma_{max} = 12,396 \text{ psi TENSION \& COMP.}$$

$$\tau_{max} = \frac{4}{3} \frac{V}{A}$$

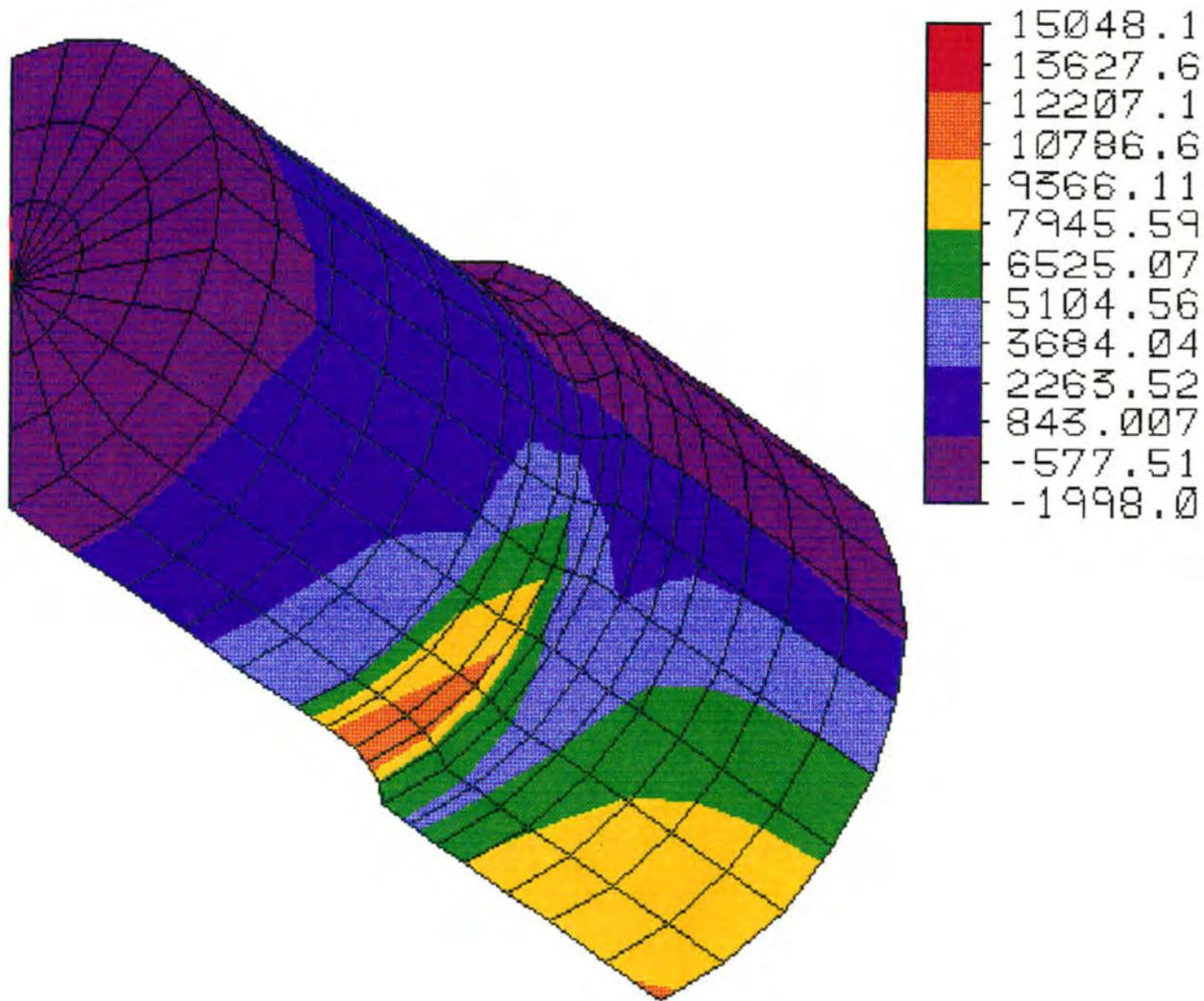
$$\tau_{max} = 2613 \text{ psi SHEAR}$$

1/4 MODEL OF THE TRUNNION SHAFT

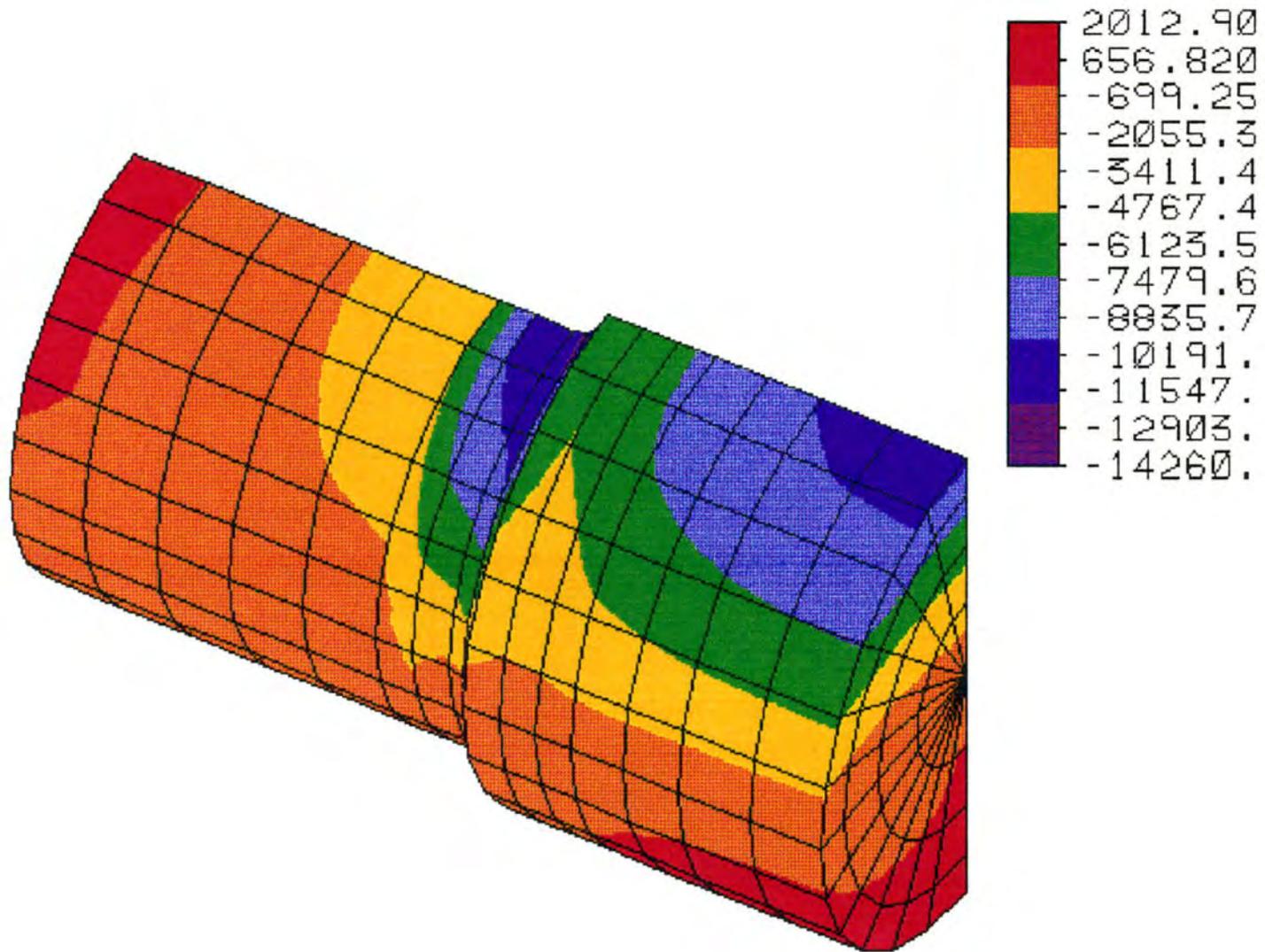




LOADING PLOT



MAXIMUM PRINCIPLE TENSILE STRESS (psi)



MAXIMUM PRINCIPLE COMPRESSIVE STRESS (psi)

## Summary

The following summary is separated into sections that correspond to the sheave's main components that were reviewed in this analysis.

### - SHEAVE -

The sheave model was created using 3-D plate elements that require a thickness input. Part of the model verification was to check this input against the various plate thickness used in the sheave design. Four load cases were developed using combinations of the derived load distributions and rotational location of the sheave during operation. The loading was verified by using a single boundary element in the vertical direction. Since loads were placed at the trunnion support to react to the main cable tray loading, this boundary element is there only to balance the model. The magnitude of this boundary element was confirmed to be within 200 pounds for all four load cases. This small amount confirmed the vertical component of the loading was balanced between the cable tray and the trunnion support.

The output of the sheave FEA model gave an estimated weight of 2 tons for a quarter model, or 8 tons for the entire sheave. The output stress plots are presented on pages 10 through 31 and are sorted by the specific load case. The following table is a summary of the peak stresses that are shown in the plots. SIG1 corresponds to the maximum principle tensile stress and SIG3 corresponds to the maximum principle compressive stress.

	-- Front Web --		Front Web Removed	
	SIG1	SIG3	SIG1	SIG3
Linear Load - No Off	3750	6340	5200	6000
- 20 Deg	3640	10,000	5420	8000
Sine Load - No Off	3050	5400	6000	6000
20 Deg	3250	10,000	6000	8500

These stresses all appear lower than the previous design. The peak stress area that was present at the top of the tear drop cutout has been significantly reduced.

### - CABLE TRAY -

The derivation of the contact loading between the rope and the sheave was used to analyze the cross section of the cable tray. A 2-D FEA model was developed and is presented on pages 32-36.

The output stress plot on page 35 indicates a peak stress of about 4000 psi tensile. On the underside of the cable tray, the maximum stress reaches a peak value of about 2400 psi. This stress is kept low mainly due to the addition of the center web. The 2-D model is based on the linear load distribution and assumes a more uniform spread of the cable pressure than the previous quarter model of the sheave does. It also includes a more detailed representation of the cable tray geometry. For these reasons, the 2-D model should represent a more accurate indication of the cable tray stresses.

- TRUNNION SHAFT -

Two methods of analysis were done for the trunnion shaft. First, a classical beam analysis was done and is presented on page 37. Next, a quarter model of the trunnion shaft was developed using 3-D, 8-noded brick elements. The output stress plot on page 40 compared well with the beam analysis. Although the bending stresses go through a complete reversal, the amount of cycles is probably going to be low when compared to traditional machine components that have similar loading and geometry characteristics.

This analysis and the presented results are based on the primary loading as being due to the cables, counterbalance, and center bridge section. If any significant additional loads do exist, they can be analyzed and superimposed with the above results.

SHEAVE LOADING

