Brom's Overturning & Torsion Loading Analysis of Short Single Shaft Foundations

This design guide illustrates the Department's recommended procedures for analyzing the depth requirements of a single drilled shaft foundation typically used in the support of traffic signals, sign structures and light towers in accordance with Article 13.6 of the Fifth Edition of the AASHTO Standard Specifications for Structural Supports for Highway Signs, Luminaires and Traffic Signals with the 2009 Interim Revisions. This design guide outlines the Department's recommended procedure for analyzing short drilled shaft foundations where the shaft's depth is governed by lateral loading and the required overturning or torsion resistance.

Overturning Analysis Procedure

Shaft foundations, supporting the structures noted above, are loaded laterally mainly by wind forces. The ability of a shaft to resist these forces is dependent on the passive pressures that develop in the soils surrounding the length and diameter of the shaft. Inherently, the passive pressure resistance is dependent on the insitu soil properties, frost depth and depth of water table.

The Department's recommended procedure for overturning analysis of a single drilled shaft foundation subject to lateral forces is based on the Brom's method as provided for in Section 13.6 of the AASHTO sign structures specifications. The procedure involves determining the passive earth pressure in each soil layer, the depth at which the shaft would tend to rotate and the total shaft depth at which the sum of the shears and moments about the shaft base equal zero. Since the shaft rotation depth cannot be directly calculated, a rotation point must be assumed and a total shaft depth calculated which results in zero shear at the base. If the sum of the moments is not also zero at this depth, the assumed point of rotation is revised and the shears, corresponding shaft depth and moments are recalculated until zero shear and moment at the base are obtained.

Hand calculations should begin with a drawing of the soil profile recognizing that an insitu soil profile may contain layers of both cohesive and granular material. Adjacent to the profile, passive pressure, and cumulative shear and moment values should be calculated at the bottom of each differing soil layer.

The following Brom's equations are used to determine the passive earth pressures.

Cohesive Passive Soil Pressure = 9(c)(D)/(tan θ + 1)

Granular Passive Soil Pressure = 3(K_p)(σ'_v)(D)

The parameters needed to solve these equations are:

c = cohesion = q_u/2 (ksf)

q_u = Unconfined compressive strength (ksf)

N = Field measured SPT blow count (blows/ft.)

D = Drilled shaft diameter (ft)

σ'_v = Effective vertical soil pressure (ksf)

σ'_v is calculated my multiplying the soil unit weight by the depth from ground surface to the mid-depth of the soil layer. The following correlations may be used for estimating the unit weight of soil (kcf):

Above water table: $\gamma_{granular} = 0.095 N_m^{0.095}$

$\gamma_{cohesive} = 0.1215 q_u^{0.095}$

Below water table: $\gamma_{granular} = 0.105 N_m^{0.07} - 0.0624$

$\gamma_{cohesive} = 0.1215 q_u^{0.095} - 0.0624$

Fill soils may be assumed to have unit weights of 0.120 kcf and 0.058 kcf above and below the water table, respectively.

t = Soil layer thickness (ft.)

K_p = Coefficient of passive earth pressure = $\cos\theta \left(\dfrac{\cos\theta + \left(\cos^2\theta - \cos^2\phi\right)^{0.5}}{\cos\theta - \left(\cos^2\theta - \cos^2\phi\right)^{0.5}} \right)$

ϕ = Angle of internal friction = $21 e \dfrac{(\ln N + 4)^2}{100}$ (degrees)

θ = Critical cross slope within a radius of 15 ft. around the shaft (degrees)

Values for q_u and N should be obtained from a soil boring log taken at, or near, to the location of the proposed foundation.

For cohesive soils, the passive pressure is neglected to a depth of 1.5 shaft diameters or the frost depth, whichever is deeper. For granular soils, passive pressure is neglected within the frost depth. Although the passive pressure is neglected in this layer, the overburden pressure of these soils is used to determine the passive pressures below that develop below these layers.

As the above passive soil pressures are considered an ultimate value, the Brom's method is considered to be an Ultimate Strength Design Method and an appropriate factor of safety (FOS) shall be used to ensure adequate resistance against overturning. For the FOS, Brom's recommended using an overload factor of 2.0 to 3.0 and an under-capacity factor of 0.7. The Department recommends using an overload factor of 2.0 when soil properties at the subject location are well established through a sufficient boring program. When soil properties are not well known, or are estimated from a distant boring, the designer should consider obtaining more accurate boring data or increasing the overload factor to 3.0. The shear and moment applied to the top of the shaft shall be increased by the FOS, which is equal to the overload factor divided by the under-capacity factor as given below:

$$\text{FOS} = \frac{\text{Overload Factor}}{\text{Under} - \text{Capacity Factor}} = \frac{2.0}{0.7} = 2.86$$

Although section 3.4 of the AASHTO manual permits a 133% increase in the allowable stress for load combinations that include wind load, this increase is not applicable to the Brom's method since it is an Ultimate Strength Design Method and is, therefore, not applied in the overturning analysis.

Torsion Analysis Procedure

Wind causes torsional rotational forces on cantilever sign trusses and traffic signal structures which require their single shaft foundations to resist these torsional forces. Torsional resistance to these forces is provided by adequate friction (in granular soils) and adhesion (in cohesive soils) against the shaft to assure it does not rotate about its longitudinal axis. The shaft depth is adequate for torsion when it engages sufficient layers of resistance equal to, or greater than, the design torsional moment. The friction or adhesion in each soil layer is calculated by the following equations:

$a_t = (c)(\alpha)$

$f_t = (\sigma'_v)(\beta)$

Where:

a_t = Adhesion of cohesive soils (*ksf*)

f_t = Friction of granular soils (*ksf*)

c = cohesion (*ksf*)

α = adhesion factor = 0.55

σ'_v = Effective vertical soil pressure to the mid-depth of soil layer (*ksf*)

$\beta = 1.5 - 0.135(h)^{1/2} \leq 1.2$

h = depth from ground surface to the mid-depth of soil layer (*ft*)

The torsional moment resistance, T_r, in a layer is calculated by multiplying the adhesion or friction by the circumference of the drilled shaft, the soil layer thickness, and the moment arm about the center of the shaft and dividing by the FOS. This is shown in the equation below:

$T_r = (a_t)\pi(D)(t)(D/2)$ / FOS (Cohesive soils)

$T_r = (f_t)\pi(D)(t)(D/2)$ / FOS (Granular Soils)

Where:

D = Diameter of drilled shaft

t = Soil layer thickness

Friction and adhesion are both neglected above the frost depth. For granular soils, the overburden pressure of the soils above the frost depth is used to determine the friction that develops against the shaft below the frost depth.

AASHTO provides no guidance on the appropriate factor of safety for torsional resistance. It has been Department practice to use a factor of safety of 1.5. The increase in allowable stress of 133% provided in Section 3.4 of the AASHTO sign structures specifications for load combinations that include wind is applicable since the torsional resistance is based on the Allowable Stress Method. Therefore, it is the Department's practice to reduce the computed torsional resistance by the following factor of Safety (FOS).

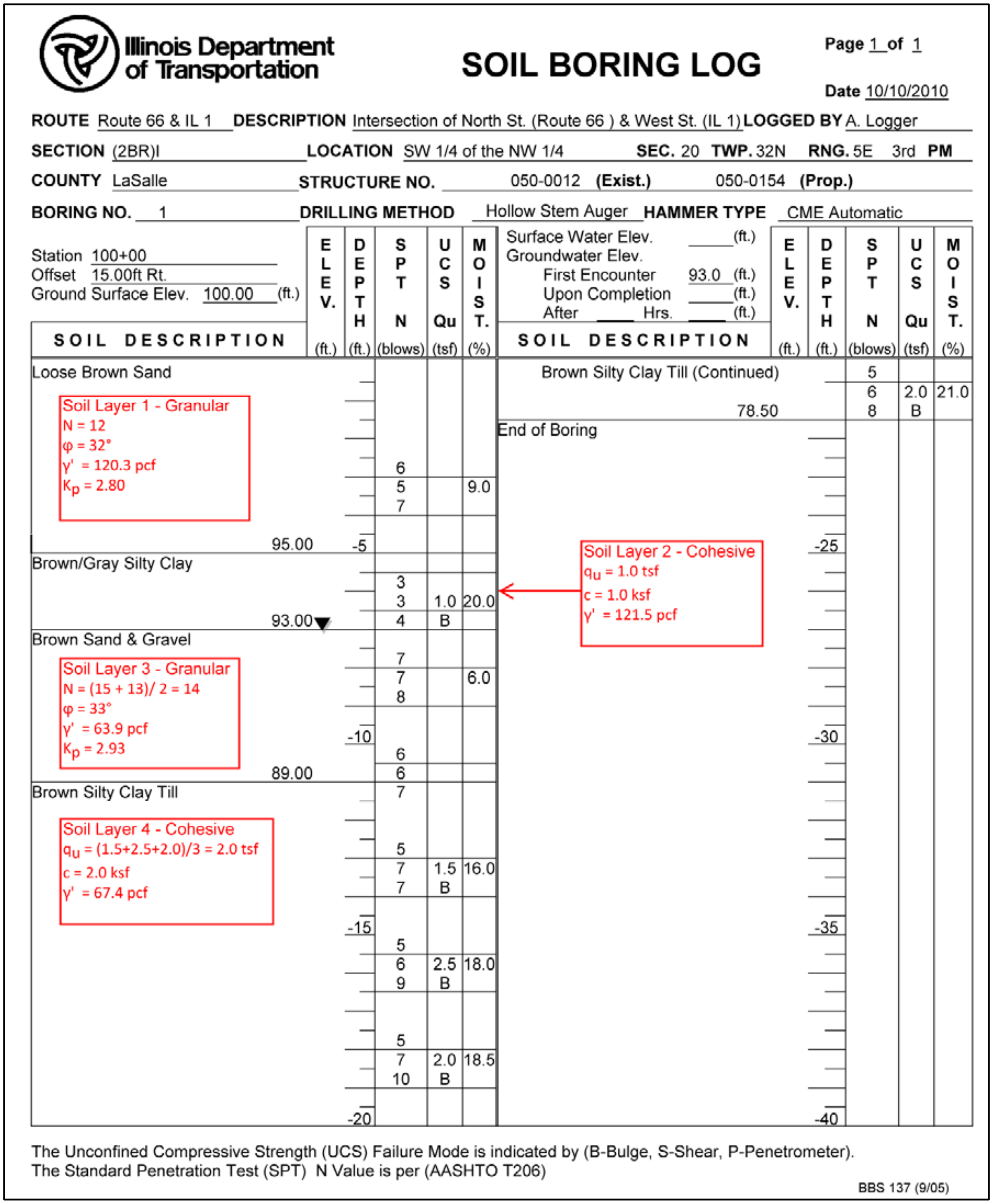
$$FOS = \frac{1.5}{1.33} = 1.13$$

The low factor of safety is deemed acceptable considering the consequences of a torsional failure would not be life threatening and there has not been a documented torsional failure since the Department adopted the current design practice over 30 years ago.

An Excel spreadsheet that performs the above calculations has been prepared to assist Engineers with conducting the overturning and torsion analyses and may be downloaded from IDOT's website.

Example for Determining the Depth of a Drilled Shaft Foundation for a Sign Structure

For a 3.5 ft diameter shaft with an applied shear, bending moment and torsional moment of 7.5 kips, 275 kip-ft and 140 kip-ft, respectively, determine the required depth to resist overturning and torsion. A soil boring, which was obtained near the location of the proposed shaft, is shown below in Figure 1.



Illinois Department of Transportation

SOIL BORING LOG

Page 1 of 1

Date 10/10/2010

ROUTE Route 66 & IL 1 DESCRIPTION Intersection of North St. (Route 66) & West St. (IL 1) LOGGED BY A. Logger

SECTION (2BR)I LOCATION SW 1/4 of the NW 1/4 SEC. 20 TWP. 32N RNG. 5E 3rd PM

COUNTY LaSalle STRUCTURE NO. 050-0012 (Exist.) 050-0154 (Prop.)

BORING NO. 1 DRILLING METHOD Hollow Stem Auger HAMMER TYPE CME Automatic

Station 100+00
Offset 15.00ft Rt.
Ground Surface Elev. 100.00 (ft.)

Surface Water Elev. ____ (ft.)
Groundwater Elev.
First Encounter 93.0 (ft.)
Upon Completion ____ (ft.)
After ____ Hrs. ____ (ft.)

SOIL DESCRIPTION	ELEV. (ft.)	DEPTH (ft.)	SPT N (blows)	UCS Qu (tsf)	MOIST. (%)
Loose Brown Sand					
			6		
			5		9.0
			7		
	95.00	-5			
Brown/Gray Silty Clay					
			3		
			3	1.0	20.0
	93.00		4	B	
Brown Sand & Gravel					
			7		
			7		6.0
			8		
		-10			
			6		
	89.00		6		
Brown Silty Clay Till			7		
			5		
			7	1.5	16.0
			7	B	
		-15			
			5		
			6	2.5	18.0
			9	B	
			5		
			7	2.0	18.5
			10	B	
		-20			

SOIL DESCRIPTION	ELEV. (ft.)	DEPTH (ft.)	SPT N (blows)	UCS Qu (tsf)	MOIST. (%)
Brown Silty Clay Till (Continued)			5		
			6	2.0	21.0
	78.50		8	B	
End of Boring					
		-25			
		-30			
		-35			
		-40			

Soil Layer 1 - Granular
$N = 12$
$\varphi = 32°$
$\gamma' = 120.3$ pcf
$K_p = 2.80$

Soil Layer 2 - Cohesive
$q_u = 1.0$ tsf
$c = 1.0$ ksf
$\gamma' = 121.5$ pcf

Soil Layer 3 - Granular
$N = (15 + 13)/ 2 = 14$
$\varphi = 33°$
$\gamma' = 63.9$ pcf
$K_p = 2.93$

Soil Layer 4 - Cohesive
$q_u = (1.5+2.5+2.0)/3 = 2.0$ tsf
$c = 2.0$ ksf
$\gamma' = 67.4$ pcf

The Unconfined Compressive Strength (UCS) Failure Mode is indicated by (B-Bulge, S-Shear, P-Penetrometer).
The Standard Penetration Test (SPT) N Value is per (AASHTO T206)

BBS 137 (9/05)

Figure 1 – Soil Boring

Overturning Analysis

First, determine the Factor of Safety (FOS) and the design shear and moment.

FOS = 2.0 / 0.7 = 2.86

$V_F = F_{applied}$ (FOS) = (7.5 *k*) x (2.86) = 21.45 *k*

$M_F = M_{applied}$ (FOS) = (275 *k-ft*) x (2.86) = 786.5 *k-ft*

Next, draw the passive soil pressure diagram based on the boring data shown in Figure 1. To draw the soil pressure diagram, a conservative depth, in this case 16', is selected and the pressures and forces are calculated over this depth. Since the upper soil layer is granular, the upper 3.5 feet of passive pressure within the frost depth is neglected. The passive soil pressure diagram is shown in figure 2.

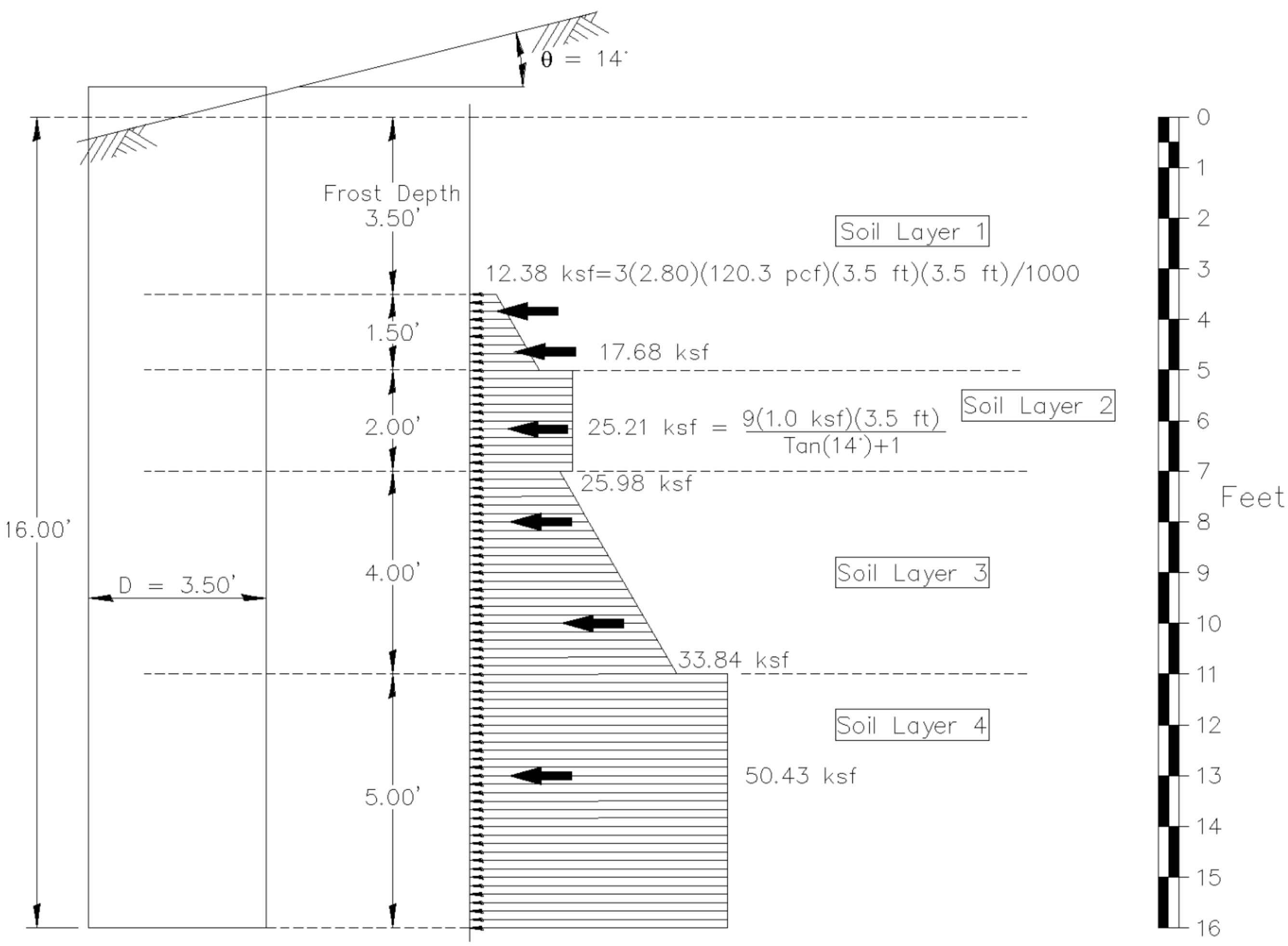


Figure 2, Soil Pressure Diagram

Finally, determine the forces in each soil layer resulting from the pressures shown in Figure 2.0. For there to be a balance of forces, a point of rotation needs to be determined. A trial point of rotation is selected, then and the total shaft depth which results in zero shear at the base is calculated. The sum of the moments at this depth is then calculated and the assumed point of rotation is revised and moments and shears are recalculated. This process continues until zero moment and shear at the base are obtained.

First Trial:

Assume a rotation depth of 11 ft. and calculate the sum of the shears and moments to the assumed point of rotation. Using a sign convention of forces to the right and clockwise moments are positive and forces to the left and counter-clockwise moments are negative, sum the shears and moments about the top of the shaft down to the assumed point of rotation.

Location	Force	ΣF_{Bottom}	Arm	Moment	ΣM_{Bottom}
Top of Shaft	21.45 *kips*	21.45	-	+786.50	+786.50
Soil Layer 2	21.45 *kips*		5.0	107.25	
	-(12.38 *k/ft*)(1.50 *ft*) = -18.57 *kips*		0.75	-13.93	
	-[(17.68 *k/ft* – 12.38 *k/ft*)/2](1.50 *ft*) = -3.98 *kips*	-1.10	0.50	-1.99	+877.83
Soil Layer 3	-1.10 *kips*		2.0	-2.20	
	-(25.21 *k/ft*)(2.00 *ft*) = -50.42 *kips*	-51.52	1.0	-50.42	+825.21
Soil Layer 4	-51.52 *kips*		4.0	-206.08	
	-(25.98 *k/ft*)(4.0 *ft*) = -103.92 *kips*		2.0	-207.84	
	-[(33.84 *k/ft* – 25.98 *k/ft*)/2](4 *ft*) = -15.72 *kips*	-171.16	1.33	-20.91	+390.38

Based on the summation of shears above and the soil pressures shown in Figure 2, the depth required to achieve zero shear below the assumed point of rotation, d, can be calculated as:

$$d = -171.16\ kips\ /\ 50.43\ k/ft = -3.394\ ft$$

Therefore, the depth to achieve zero shear is 3.394 feet below the assumed point of rotation of 11 feet, or a shaft depth of 14.394 feet. Next, compute the sum of the moments at this depth.

Location	Force	ΣF_{Bottom}	Arm	Moment	ΣM_{Bottom}
Soil Layer 5	-171.16 *kips*		3.394	-580.92	
	(50.43 *k/ft*)(3.394 *ft*) = 171.160 *kips*	0.0	1.697	+290.46	+99.92

A positive moment at this depth indicates that the assumed point of rotation is too high and must be lowered. Try a depth of 11.5 feet.

Second Trial:

Assume a rotation depth of 11.5 feet and calculate the depth that results in zero shear.

Location	Force	ΣF_{Bottom}	Arm	Moment	ΣM_{Bottom}
Top of Shaft	21.45 *kips*	21.45	-	+786.50	+786.50
Soil Layer 2	21.45 *kips*		5.0	107.25	
	-(12.38 *k/ft*)(1.50 *ft*) = -18.57 *kips*		0.75	-13.93	
	-[(17.68 *k/ft* – 12.38 *k/ft*)/2](1.50 *ft*) = -3.98 *kips*	-1.10	0.50	-1.99	+877.83
Soil Layer 3	-1.10 *kips*		2.0	-2.20	
	-(25.21 *k/ft*)(2.00 *ft*) = -50.42 *kips*	-51.52	1.0	-50.42	+825.21
Soil Layer 4	-51.52 *kips*		4.0	-206.08	
	-(25.98 *k/ft*)(4.0 *ft*) = -103.92 *kips*		2.0	-207.84	
	-[(33.84 *k/ft* – 25.98 *k/ft*)/2](4 *ft*) = -15.72 *kips*	-171.16	1.33	-20.91	+390.38
Soil Layer 5	-171.16 *kips*		0.50	-85.58	
	-(50.43 *k/ft*)(0.5 *ft*) = -25.22 *kips*	-196.38	0.25	-6.31	+298.49

The depth required to achieve zero shear below the assumed point of rotation, d, is then:

d = -196.38 *kips* / 50.43 *k/ft* = -3.894 *ft*

Therefore, the depth to achieve zero shear is 3.894 feet below the assumed point of rotation of 11.5 feet, or a shaft depth of 15.394 feet. Next, compute the sum of the moments at this depth.

Location	Force	ΣF_{Bottom}	Arm	Moment	ΣM_{Bottom}
Soil Layer 5	-196.38 *kips*		3.894	-764.70	
	(50.43 *k/ft*)(3.894 *ft*) = 196.37 *kips*	-.01	1.947	+382.33	-83.88

A negative moment at this depth indicates that the assumed point of rotation is too low and must be raised. Linear interpolation of the moment between the first two trials results in a depth to the point of rotation of 11.272 feet. Therefore, try a depth of 11.272 feet.

Third Trial:

Assume a rotation depth of 11.272 feet and calculate the depth that results in zero shear.

Location	Force	ΣF_{Bottom}	Arm	Moment	ΣM_{Bottom}
Top of Shaft	21.45 *kips*	21.45	-	+786.50	+786.50
Soil Layer 2	21.45 *kips*		5.0	107.25	
	-(12.38 *k/ft*)(1.50 *ft*) = -18.57 *kips*		0.75	-13.93	
	-[(17.68 *k/ft* – 12.38 *k/ft*)/2](1.50 *ft*) = -3.98 *kips*	-1.10	0.50	-1.99	+877.83
Soil Layer 3	-1.10 *kips*		2.0	-2.20	
	-(25.21 *k/ft*)(2.00 *ft*) = -50.42 *kips*	-51.52	1.0	-50.42	+825.21
Soil Layer 4	-51.52 *kips*		4.0	-206.08	
	-(25.98 *k/ft*)(4.0 *ft*) = -103.92 *kips*		2.0	-207.84	
	-[(33.84 *k/ft* – 25.98 *k/ft*)/2](4 *ft*) = -15.72 *kips*	-171.16	1.33	-20.91	+390.38
Soil Layer 5	-171.16 *kips*		0.272	-46.56	
	-(50.43 *k/ft*)(0.272 *ft*) = -13.72 *kips*	-184.88	0.136	-1.87	+341.95

The depth required to achieve zero shear below the assumed point of rotation, d, is then:

d = -184.88 *kips* / 50.43 *k/ft* = -3.666 *ft*

Therefore, the depth to achieve zero shear is 3.666 feet below the assumed point of rotation of 11.272 feet, or a shaft depth of 14.938 feet. Now, compute the sum of the moments at this depth.

Location	Force	ΣF_{Bottom}	Arm	Moment	ΣM_{Bottom}
Soil Layer 5	-184.88 *kips*		3.666	-677.77	
	(50.43 *k/ft*)(3.666 *ft*) = 184.88 *kips*	0.0	1.833	+338.89	+3.07

Shear and moment are both approximately zero, therefore no further refinement is necessary.

The following is a summary of the above design procedure:

$Depth_{Min}$ = 14.938 *ft*

V_{max} = 184.88 *kips*

M_{Max} = 877.83 *k-ft*

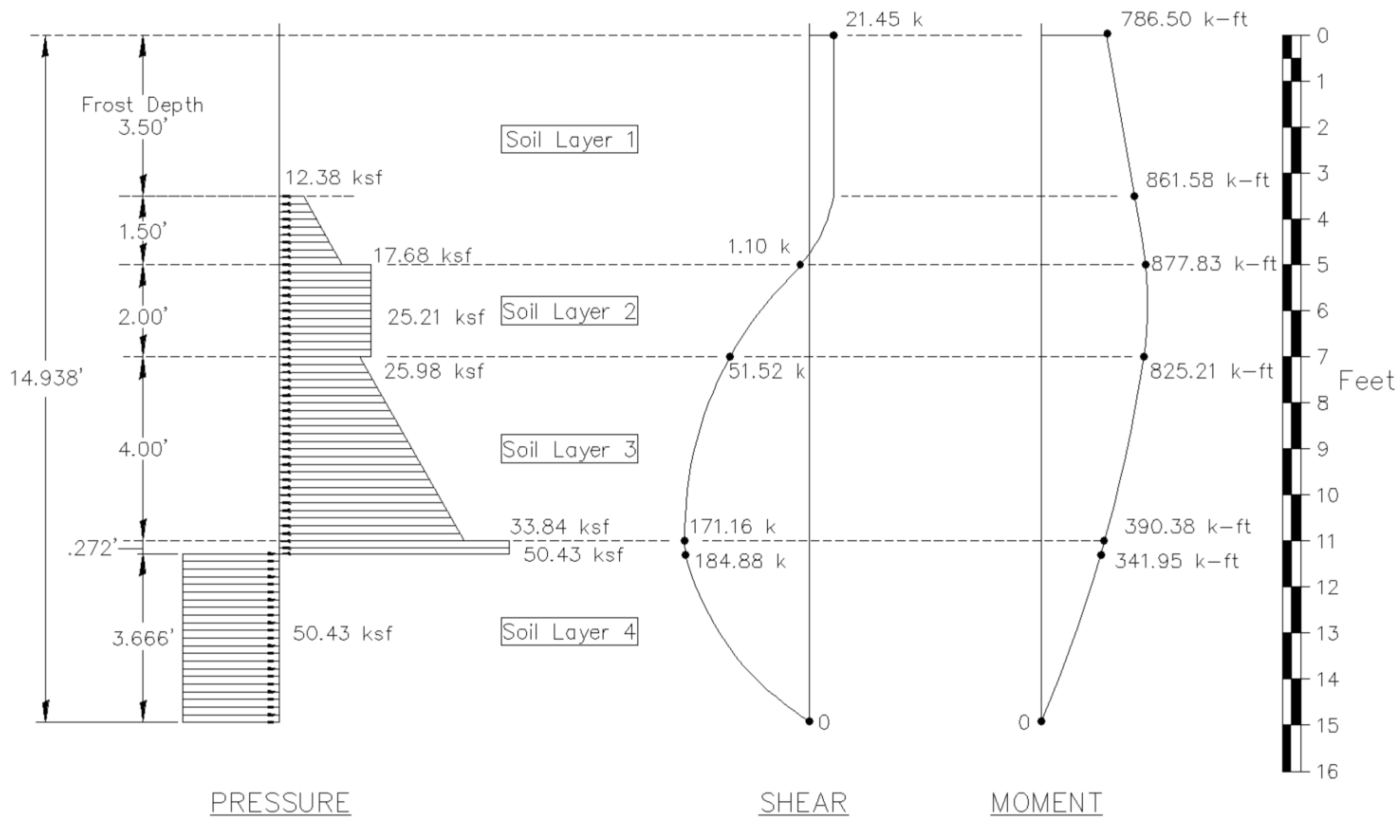


Figure 3.0, Final Design Soil Pressure and Shear Diagrams

Torsion Analysis

Sum the torsional moment resistances for each layer until the total resistance equals the design torsional moment. The equations are chosen based on the soil descriptions and strength data.

The design torsional moment, T, equals 140.0 *kip-ft.* Compute the torsional resistance, T_r, for each differing soil layer.

$\beta_{Layer\ \#1}$ = 1.5 - .0135(3.5 + 1.5 / 2)^{½} = 1.222 ≤ 1.2

$f_{t\ Layer\ \#1}$ = (0.1203 *kcf*)(4.25 *ft*)(1.2) = 0.614 *ksf*

$T_{r\ Layer\ \#1}$ = (0.614 *ksf*)π(3.5 *ft*)(1.50 *ft*) (3.50 *ft* / 2) / 1.13 = 15.67 *kip-ft*

ΣT = 140.0 – 15.67 = 124.33 *kip-ft*

$a_{t\ Layer\ \#2}$ = (1.0 *ksf*)(0.55) = 0.55 *ksf*

$T_{r\ Layer\ \#2}$ = (0.55)π(3.5 *ft*)(2.0 *ft*)(3.50 *ft* / 2) / 1.13 = 18.73 *kip-ft*

ΣT = 124.33 – 18.73 = <u>105.60</u> *kip-ft*

$\beta_{Layer\ \#3}$ = 1.5 - .0135(5.0 + 2.0 + 4.0/2)^{½} = 1.095 ≤ 1.2

$f_{t\ Layer\ \#3}$ = (0.1203 *kcf*)(5.0 *ft*)+(0.1215)(2.0 *ft*)+(0.0639)(2.0 *ft*) = 0.972 *ksf*

$T_{r\ Layer\ \#3}$ = (0.972 *ksf*)(1.095)π(3.5 *ft*)(4.0 *ft*)(3.5 *ft* / 2) / 1.13 = 72.50 *kip-ft*

ΣT = 105.60 – 72.50 = <u>33.10</u> *kip-ft*

Determine the thickness, t, required in soil layer 4 to achieve a torsional resistance of 33.10 *kip-ft.*

$a_{t\ Layer\ \#4}$ = (2.0 *ksf*)(0.55) = 1.10 *ksf*

$T_{r\ Layer\ \#4}$ = (1.10)π(3.5 *ft*)(t)(3.5 *ft* / 2) / 1.13 = 33.10 *kip-ft*

Solving for t,

t = (33.10 *kip-ft*)(1.13) / (1.10)π(3.5 *ft*)(3.5 *ft* / 2) = 1.77 *ft*

Therefore, the total depth of shaft required to resist the applied torsional moment is 12.77 feet.

Since the depth required to resist overturning is greater than that required to resist torsion, a depth of say 15 ft. should be used in plan documents.

Relevant References

AASHTO "*Standard Specifications for Structural Supports for Highway Signs, Luminaires, and Traffic Signals",* Fifth Edition 2009, Section 13.6 pp. 13-2 to 13-6.

Colorado DOT, Report No. CDOT-DTD-R-2004-8, "*Drilled Shaft Design for Sound Barrier Walls, Signs and Signals",* Final Report 2004, pp. C-21 to C-22.

University of Florida, *"Determining The Optimum Depth of Drilled Shafts Subject to Combined Torsion and Lateral Loads in Saturated Sand from Centrifuge Testing",* Master's Degree Thesis by ZHIHONG HU, 2003, pp. 20 - 21

FHWA Publication No. FHWA-SA-91-048, "*COM624P-Laterally Loaded Pile Analysis Program for the Microcomputer*", Part II: Background, Chapter 3, pp 357 to 361.

Brown, Dan A. and Turner, John P. and Castelli, Raymond J. FHWA Report No. NHI-10-016. "Drilled Shafts: Construction Procedures and LRFD Design Methods". NHI Course No. 132014.