Illinois Department of Transportation
Office of Highways Project Implementation / Bureau of Bridges & Structures
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All IDOT Design Guides have been updated to reflect the release of the 2017 AASHTO LRFD Bridge Design Specification, 8^{th} Edition. The following is a summary of the major changes that have been incorporated into the PPC Deck Beam Design Guide.

- The equation for the concrete modulus of elasticity, E_c, has been modified.

3.5 LRFD PPC Deck Beam Design

This design guide focuses on the LRFD design of PPC Deck Beams. The design procedure is presented first followed by an example. All Article and Equation references are to the LRFD code unless noted otherwise.

There are twelve standard beam cross sections supported by the Department. These are 11x48, 11x52, 17x36, 17x48, 21x36, 21x48, 27x36, 27x48, 33x36, 33x48, 42x36 and 42x48. Standard strand patterns for LRFD designs have been developed and are provided in Bridge Manual (BM) Tables 3.5.3-1 thru 3.5.3-6. All other reinforcement details (shear reinforcement, splitting steel, top slab reinforcement, etc.) have been standardized and are shown on the base sheets. Aids for detailing the end blocks and designing lifting loops are shown in BM Figures 3.5.9-3 and 3.5.9-4.

The main items a designer has to calculate are distribution factors, moment envelopes, prestress losses, temporary stresses, service stresses and resisting moment capacities; all of which are used to determine the required strand pattern.

The sign convention used in the examples was noted by labeling negative results with “tension” and positive results with “compression” unless otherwise noted.

LRFD Design Procedure, Equations and Outline

Transverse Ties

Transverse ties are used on all deck beams except the 11 inch deep sections which are too shallow to accommodate the ties. The 11 inch beams rely mainly on the dowel rods at fixed substructure units and side retainers at expansion substructure units in lieu of the ties. For 17 through 42 inch beams transverse ties are placed along the skew throughout the span with a uniform spacing. The beam base sheets show the standard configuration for a transverse tie diaphragm. The number of ties is determined by the following formula:

$$\text{Number ties} = \left(\frac{L}{25} - 1\right) \geq 1 \quad \text{rounded up to the nearest integer}$$

Where:

L = span of beam (ft.)

Dead Loads

Calculate the dead loads on a single beam such as the beam self weight, wearing surface, future wearing surface, parapets or railing and any other dead loads on the bridge and group them into their appropriate types DC and DW. Distribution of railings or parapets shall be spread over three beams. Sidewalks and medians shall be distributed over the number of beams they cover plus one or two depending on if there are beams on both sides.

The beam self weight is non-uniform for all beams except the 11 inch sections. The weights for both the solid and voided sections of each beam size can be found in BM Figure 3.5.9-4. Typically a designer would use the net section uniform load in conjunction with a series of point loads that would represent the solid portions at the transverse tie diaphragms and end blocks. The solid part at the end blocks can be ignored for moment calculations however they should be included for the lifting loop and substructure design. See Figure 1 for the loading diagrams used to calculate the moment due to beam self weight with one to four transverse tie diaphragms.

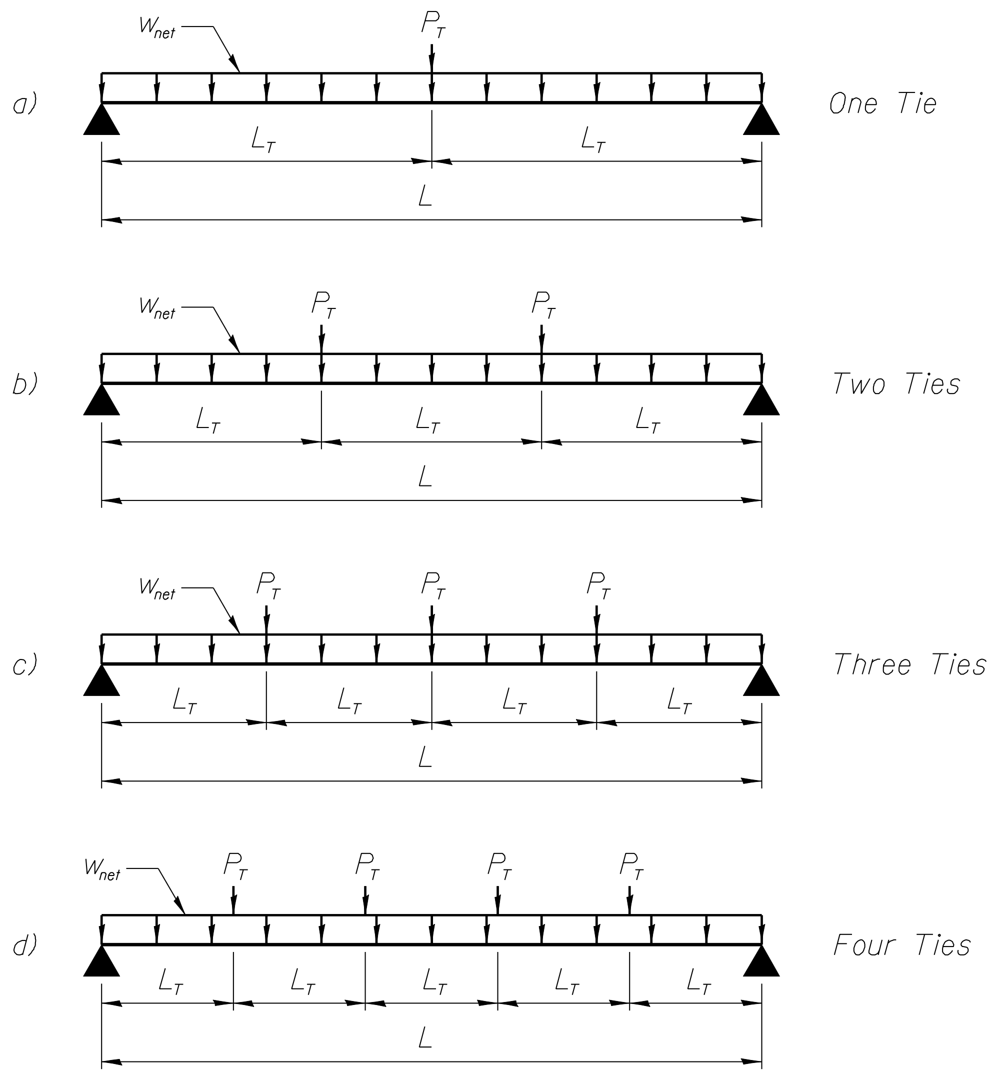


Figure 1

Formulas for Moment due to Beam Self Weight

a) $M_b = \dfrac{w_{net}L^2}{8} + \dfrac{P_T L}{4}$

b) $M_b = \dfrac{w_{net}L^2}{8} + P_T L_T$

c) $M_b = \dfrac{w_{net}L^2}{8} + 2P_T L_T$

d) $M_b = \dfrac{w_{net}L^2}{8} + 3P_T L_T$

In which:

$$P_T = \frac{2\text{ ft.}}{\cos(\text{skew})}\left(w_{solid} - w_{net}\right)$$

Where:

M_b = bending moment due to beam self weight (kip-ft.)

w_{net} = weight per unit length of the section of the beam with voids (kip/ft.)

w_{solid} = weight per unit length of the section of the beam without voids (kip/ft.)

P_T = weight of transverse tie diaphragm (kips)

L_T = transverse tie spacing (ft.)

L = span of beam (ft.)

Section Properties

Deck beams are always designed as non-composite sections even if a reinforced concrete overlay is used. The section properties for each beam section are provided in BM Table 3.5.4-1.

Modulus of Elasticity

The modulus of elasticity for concrete shall be determined from the following formulas:

$$E_{ci} = 120{,}000K_1w_c^{2.0}f'^{0.33}_{ci} \qquad \text{(Eq. 5.4.2.4-1)}$$

$$E_c = 120{,}000K_1w_c^{2.0}f'^{0.33}_{c}$$

Where:

E_{ci} = modulus of elasticity of concrete at transfer (ksi)

E_c = modulus of elasticity of concrete (ksi)

K_1 = aggregate modification factor, taken as 1.0

w_c = unit weight of concrete, as per Table 3.5.1-1, this is taken as 0.140 + 0.001f'_c, or 0.146 kcf for 6 ksi concrete. However, it is typical to use either 0.150 kcf or 0.145 kcf.

f'_{ci} = specified compressive strength of concrete at time of initial loading or prestressing (ksi)

f'_c = specified compressive strength of concrete for use in design (ksi)

Distribution Factors for Moment

Live load moments shall be distributed according to Article 4.6.2.2. See Table 4.6.2.2.2b-1 under "Concrete Beams used in Multibeam Decks", cross section (g) "connected only enough to prevent relative vertical displacement at the interface". It should be noted that the department does not use the distribution factors associated with cross section (f) for reinforced concrete overlays. The Department also does not consider exterior beam distribution factors in Table 4.6.2.2.2d-1 or reduction of distribution factors in Table 4.6.2.2.2e-1. The distribution equation below can be used for both the final and staged construction cases.

Regardless of the number of loaded lanes the distribution factor equals:

$$g = \frac{S}{D}$$

In which:

$$D = 11.5 - N_L + 1.4N_L(1 - 0.2C)^2 \quad \text{when } C \le 5$$

$$11.5 - N_L \quad \text{when } C > 5$$

$$C = K\left(\frac{W}{L}\right) \le K$$

$$K = \sqrt{\frac{(1+\mu)I}{J}}$$

Where:

g = distribution factor

S = beam spacing (ft.)

D = width of distribution per lane (ft.)

N_L = number of design lanes as specified in Article 3.6.1.1.1

C = stiffness parameter

K = constant for different types of construction

W = edge-to-edge width of bridge (ft.)

L = span of beam (ft.)

I = moment of inertia (in.^{4})

μ = Poisson's ratio, assumed to be 0.2

J = St. Venant's torsional inertia (in.^{4})

The distribution factor for fatigue evaluation equals the factor calculated above divided by the multi presence factor for one lane. See BM Section 3.3.1.

Design Moment

Since deck beam bridges in Illinois are typically not designed as continuous structures the moments can be calculated by hand or by using software written for the task.

Strand Pattern Selection

The planning selection charts located in BM Section 2.3.6.1.2 can be used to determine a trial strand pattern. The properties of the trial strand pattern can be found in BM Tables 3.5.3-1 through 3.5.3-6.

Prestress Losses *(5.9.3)*

Calculate prestress losses.

<u>Total Loss of Prestress</u> (5.9.3.1)

$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$ (Eq. 5.9.3.1-1)

Where:

Δf_{pT} = total loss (ksi)

Δf_{pES} = loss in prestressing steel due to elastic shortening (instantaneous losses) (ksi)

Δf_{pLT} = losses due to long term shrinkage and creep of concrete, and relaxation of the steel (ksi)

Instantaneous Losses (5.9.3.2)

It is the Department's policy to consider elastic losses only. Elastic gains are not considered.

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp} \qquad \text{(Eq. 5.9.3.2.3a-1)}$$

In which:

$$f_{cgp} = \frac{F_t}{A} + \frac{F_t e^2}{I} - \frac{M_b(12)e}{I}$$

Assume F_t equals 90 percent of F_i for first iteration (C5.9.3.2.3a)

F_t = $0.9(F_i)$

F_i = $A_{ps}(f_{pbt})$

Calculate Δf_{pES} and verify assumption

$$\text{Assumption} \approx \frac{(f_{pbt} - \Delta f_{pES})}{f_{pbt}}$$

Reiterate if necessary

Where:

Δf_{pES} = loss in prestressing steel due to elastic shortening (ksi)

E_p = modulus of elasticity of prestressing steel (ksi)

E_{ci} = modulus of elasticity of concrete at transfer (ksi)

f_{cgp} = concrete stress at the center of gravity of prestressing tendons due to prestressing force immediately after transfer and the self-weight of the member at the section of maximum moment (center) (ksi)

F_t = total prestressing force immediately after transfer (kips)

F_i = total prestressing force prior to transfer (kips)

e = eccentricity of centroid of strand pattern from NA of beam (in.)

M_b = bending moment due to beam self weight (kip-ft.)

A = area of beam (in.^{2})

I = moment of inertia of beam (in.^{4})

A_{ps} = total area of prestressing steel (in.^{2})

f_{pbt} = stress in prestressing steel immediately prior to transfer (ksi)

Time Dependent Losses (5.9.3.3)

The AASHTO LRFD Bridge Design Specifications contains two methods for calculating time dependent losses: the "Approximate" method in Article 5.9.3.3 and the "Refined" method in Article 5.9.3.4. The approximate method is fairly straightforward and requires only basic information that is readily available to the designer. The refined method is much more rigorous; the equations are difficult to follow; and information is required that the designer does not have direct control over. Some of these variables are the time the strands are released, the time the deck is placed and the actual concrete strength at release. These variables require the designer to use a best-estimate guess, yet these variables can impact the results significantly. Article 5.9.5.3 provides further clarification for the application of the approximate and refined methods. In particular it specifies that the refined method shall be used for members that are non-composite which would appear applicable for deck beam structures. However, the Department has evaluated the specifications and our details and developed the following policy:

Deck beams with a 5 inch concrete wearing surface shall use the approximate method for time dependent losses. The Department believes the concrete wearing surface behaves as a partially composite section with the beams even though it's not considered to be partially composite for strength calculations. This logic is based upon the required textured broom finish on the top surface of the beam, the D(E) bars that protrude from the tops of the exterior beams for F-shaped barriers and Type SM railings, and because there have been no signs of separation between the beams and the CWS on our inventory inspections.

Deck beams with no wearing surface or a bituminous wearing surface shall also use the approximate method for time dependent losses. However, in order to satisfy the spirit of the code, a correction factor shall be added to the results obtained from the approximate method to more closely match the intent of the refined method. The correction factors are as follows:

Deck beam depth (in.)	Correction factor (ksi)
11	4.5
17	4.5
21	4.5
27	4.5
33	3.5
42	2.5

For example: If the designer calculates an approximate time dependent loss of 20 ksi for a 17 inch beam, a correction factor of 4.5 ksi should be added to make it 24.5 ksi.

$$\Delta f_{pLT} = 10.0\frac{f_{pbt}A_{ps}}{A}\gamma_h\gamma_{st} + 12.0\gamma_h\gamma_{st} + \Delta f_{pR} \qquad \text{(Eq. 5.9.3.3-1)}$$

In which:

$$\gamma_h = 1.7 - 0.01H \qquad \text{(Eq. 5.9.3.3-2)}$$

$$\gamma_{st} = \frac{5}{(1 + f'_{ci})} \qquad \text{(Eq. 5.9.3.3-3)}$$

Where:

Δf_{pLT} = losses due to long term shrinkage and creep of concrete, and relaxation of the steel (ksi)

f_{pbt} = stress in prestressing steel immediately prior to transfer (ksi)

A_{ps} = total area of prestressing steel (in.^{2})

A = area of beam (in.^{2})

γ_h = correction factor for relative humidity

γ_{st} = correction factor for specified concrete strength at time of prestress transfer

Δf_{pR} = estimate of relaxation loss taken as 2.4 ksi for low relaxation strands (ksi)

H = relative humidity, assumed to be 70% in Illinois (%)

f'_{ci} = specified compressive strength of concrete at time of initial loading or prestressing (ksi)

Temporary Stresses *(5.9.2.3)*

Temporary stresses are checked immediately after the release of the strands when the concrete strength, f'_{ci}, is weakest. The force in the strands is taken to be the prestressing force immediately after transfer, F_t.

There are three support conditions to consider during this time frame. The first occurs when the strands are released and the beam is still setting on the prestressing bed. The second occurs when lifting the beam out of the prestressing bed. The third occurs when placing the beam in temporary storage at the fabrication plant. Theoretically, all three of these conditions could take place while the concrete is most vulnerable, however the third condition will govern for deck beams if current IDOT policies are followed. Therefore this is the only condition checked.

For this case, the stresses need to be checked in two locations, which are at the center of the temporary supports and at the center of the beam.

See Figure 2 for the support and loading diagram used to calculate the dead load moments for checking temporary stresses. The weights of transverse tie diaphragms are conservatively ignored.

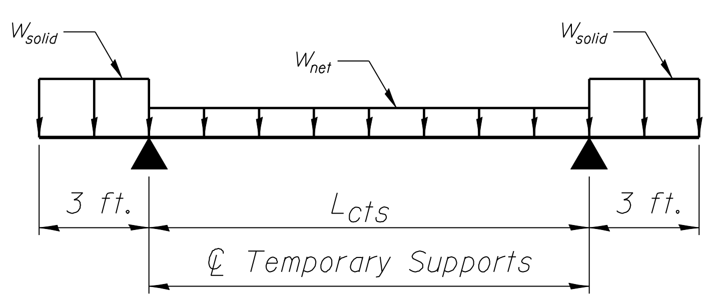


Figure 2

@ Temporary Supports:

$$M_{bts} = \frac{w_{solid}(3\text{ ft.})^2}{2}$$

@ Center:

$$M_{bts} = \frac{w_{net}(L_{cts})^2}{8} - \frac{w_{solid}(3\text{ ft.})^2}{2}$$

Where:

M_{bts} = bending moment due to beam self weight with supports at temporary locations (kip-ft.)

w_{net} = weight per unit length of the section of the beam with voids (kip/ft.)

w_{solid} = weight per unit length of the section of the beam without voids (kip/ft.)

L_{cts} = length from center to center of temporary supports (ft.)

<u>Temporary Stress Limits for Concrete</u> (5.9.2.3.1)

Compression:

$0.65f'_{ci}$ (5.9.2.3.1a)

Tension:

$0.24\lambda\sqrt{f'_{ci}}$ (Table 5.9.2.3.1b-1)

Where:

f'_{ci} = specified compressive strength of concrete at time of initial loading or prestressing (ksi)

λ = concrete density modification factor, taken as 1.0 for normal-weight concrete (5.4.2.8)

<u>Calculate Temporary Stresses</u>

@ Temporary Supports:

$$f_t = \frac{F_t}{A} - \frac{F_t e}{S_t} - \frac{M_{bts}(12)}{S_t}$$

$$f_b = \frac{F_t}{A} + \frac{F_t e}{S_b} + \frac{M_{bts}(12)}{S_b}$$

@ Center:

$$f_t = \frac{F_t}{A} - \frac{F_t e}{S_t} + \frac{M_{bts}(12)}{S_t}$$

$$f_b = \frac{F_t}{A} + \frac{F_t e}{S_b} - \frac{M_{bts}(12)}{S_b}$$

In which:

$F_t = A_{ps}(f_{pbt} - \Delta f_{pES})$

Where:

f_t = concrete stress at the top fiber of the beam (ksi)

f_b = concrete stress at the bottom fiber of the beam (ksi)

F_t = total prestressing force immediately after transfer (kips)

A = area of beam (in.^{2})

e = eccentricity of centroid of strand pattern from NA of beam (in.)

S_t = non-composite section modulus for the top fiber of the beam (in.^{3})

S_b = non-composite section modulus for the bottom fiber of the beam (in.^{3})

M_{bts}= bending moment due to beam self weight with supports at temporary locations (kip-ft.)

A_{ps} = total area of prestressing steel (in.^{2})

f_{pbt} = stress in prestressing steel immediately prior to transfer (ksi)

Δf_{pES} = loss in prestressing steel due to elastic shortening (ksi)

Service Stresses *(5.9.2.3.2)*

Service limit state stresses are checked for the beam in its final placement in the structure. The concrete strength is equal to f'_c and the force in the strands is equal to F_S.

Compressive service stresses are calculated for the two applicable Service I load combinations given in Table 5.9.2.3.2a-1. For simplicity, these have been given the nomenclature (a) and (b) in this design guide. Tensile service stresses are calculated for the one applicable Service III load combination given in Table 5.9.2.3.2b-1. The factored

Service I and Service III load combinations are found in Table 3.4.1-1 and the load factors have been applied to the equations shown below.

Service Stress Limits for Concrete after Losses (5.9.2.3.2)

Compression (For Service I load combination):

$0.60\phi_w f'_c$ (a) (Table 5.9.2.3.2a-1)

$0.45f'_c$ (b) (Table 5.9.2.3.2a-1)

Tension (For Service III load combination):

$0.19\lambda\sqrt{f'_c}$ (Table 5.9.2.3.2b-1)

Where:

f'_c = specified compressive strength of concrete for use in design (ksi)

ϕ_w = hollow column reduction factor, equals 1.0 for standard IDOT sections

λ = concrete density modification factor, taken as 1.0 for normal-weight concrete (5.4.2.8)

Calculate Service Stresses

Service stresses are calculated from the following equations:

@ Center:

$$f_t = \frac{F_S}{A} - \frac{F_S e}{S_t} + \frac{(M_{DC1} + M_{DW1} + M_{LL+IM})(12)}{S_t} \quad \text{(a)}$$

$$f_t = \frac{F_S}{A} - \frac{F_S e}{S_t} + \frac{(M_{DC1} + M_{DW1})(12)}{S_t} \quad \text{(b)}$$

$$f_b = \frac{F_s}{A} + \frac{F_s e}{S_b} - \frac{(M_{DC1} + M_{DW1})(12)}{S_b} - (0.8)\frac{M_{LL+IM}(12)}{S_b}$$

In which:

$F_s = A_{ps}(f_{pbt} - \Delta f_{pT})$

Where:

f_t = concrete stress at the top fiber of the beam (ksi)

f_b = concrete stress at the bottom fiber of the beam (ksi)

F_S = total prestressing force after all losses (kips)

A = area of beam (in.^{2})

e = eccentricity of centroid of strand pattern from NA of beam (in.)

M_{DC1} = unfactored non-composite dead load moment of structural components and nonstructural attachments (kip-ft.)

M_{DW1} = unfactored non-composite dead load moment of wearing surfaces and utilities (kip-ft.)

M_{LL+IM} = unfactored live load moment (HL-93) plus dynamic load allowance (kip-ft.)

S_t = non-composite section modulus for the top fiber of the beam (in.^{3})

S_b = non-composite section modulus for the bottom fiber of the beam (in.^{3})

A_{ps} = total area of prestressing steel (in.^{2})

f_{pbt} = stress in prestressing steel immediately prior to transfer (ksi)

Δf_{pT} = total loss (ksi)

Fatigue Stresses *(5.5.3.1)*

The compressive stress due to the Fatigue I load combination and one-half the sum of effective prestress and permanent loads shall not exceed the limit shown below. The section properties used for calculating the compressive stress are determined based on whether the tensile stress limit shown below is exceeded. The tensile stress is calculated using the Fatigue I load combination plus effective prestress and permanent loads.

Fatigue Stress Limits for Concrete after Losses (5.5.3.1)

Compression:

$$0.40f'_c$$

Tension limit for determination cracking of section:

$$\text{Uncracked} \leq 0.095\sqrt{f'_c} \leq \text{Cracked}$$

Where:

f'_c = specified compressive strength of concrete for use in design (ksi)

<u>Calculate Fatigue Stresses</u>

Fatigue stress is calculated from the following equation:

@ Center:

$$f_t = 0.5\left[\frac{F_S}{A} - \frac{F_S e}{S_t} + \frac{(M_{DC1} + M_{DW1})(12)}{S_t}\right] + 1.5\frac{M_{FL+IM}(12)}{S_t}$$

Tension stress is calculated from the following equation:

(This stress is only used to determine whether the section is considered cracked or uncracked for fatigue evaluation only)

@ Center:

$$f_b = \frac{F_s}{A} + \frac{F_s e}{S_b} - \frac{(M_{DC1} + M_{DW1})(12)}{S_b} - (1.5)\frac{M_{FL+IM}(12)}{S_b}$$

Where:

f_t = concrete stress at the top fiber of the beam (ksi)

f_b = concrete stress at the bottom fiber of the beam (ksi)

f'_c = specified compressive strength of concrete for use in design (ksi)

F_S = total prestressing force after all losses (kips)

A = area of beam (in.^{2})

e = eccentricity of centroid of strand pattern from NA of beam (in.)

M_{DC1}= unfactored non-composite dead load moment of structural components and nonstructural attachments (kip-ft.)

M_{DW1}= unfactored non-composite dead load moment of wearing surfaces and utilities (kip-ft.)

M_{FL+IM}=unfactored fatigue live load moment plus dynamic load allowance (kip-ft.)

S_t = non-composite section modulus for the top fiber of the beam (in.^{3})

S_b = non-composite section modulus for the bottom fiber of the beam (in.^{3})

Flexural Resistance *(5.6.3)*

The design procedure for the flexural resistance of a prestressed concrete member is outlined below. Please note that it is the Department's policy to not utilize non-prestressed tension reinforcement.

Strength I Moment

$M_u = 1.25(M_{DC1}) + 1.5(M_{DW1}) + 1.75(M_{LL+IM})$ (Table 3.4.1-1)

Impact shall be taken as 33% (Table 3.6.2.1-1). Engineering judgment may be used when determining the value of the "η" load modifiers specified in Article 1.3.2. As these are normally assumed to be 1.0 in standard bridges and therefore do not affect the design, they will not be addressed any further in this design guide.

Factored Flexural Resistance

$$M_r = \phi M_n$$

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)\left(\frac{1}{12}\right) \quad \text{rectangular} \quad \text{(Eq. 5.6.3.2.2-1)}$$

$$M_n = \left[A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85 f'_c (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right)\right]\left(\frac{1}{12}\right) \quad \text{flanged} \quad \text{(Eq. 5.6.3.2.2-1)}$$

In which:

$$a = \beta_1 c$$

$$c = \frac{A_{ps} f_{pu}}{\alpha_1 f'_c \beta_1 b + k A_{ps} \frac{f_{pu}}{d_p}} \quad \text{rectangular} \quad \text{(Eq. 5.6.3.1.1-4)}$$

$$c = \frac{A_{ps} f_{pu} - 0.85 f'_c (b - b_w) h_f}{\alpha_1 f'_c \beta_1 b_w + k A_{ps} \frac{f_{pu}}{d_p}} \quad \text{flanged} \quad \text{(Eq. 5.6.3.1.1-3)}$$

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) \quad \text{If } f_{pe} \geq 0.5 f_{pu} \quad \text{(Eq. 5.6.3.1.1-1)}$$

$$f_{pe} = f_{pu} - \Delta f_{pT}$$

$$\phi = 0.75 \leq 0.75 + \frac{0.25(\varepsilon_t - \varepsilon_{cl})}{(\varepsilon_{tl} - \varepsilon_{cl})} \leq 1.0 \quad \text{(Eq. 5.5.4.2.1-1)}$$

$\beta_1 = 0.65 \leq 0.85 - 0.05\left(f'_c - 4.0\right) \leq 0.85$ (5.7.2.2)

Where:

M_u = factored moment at the section (kip-ft.)

M_r = factored flexural resistance of a section in bending (kip-ft.)

M_n = nominal flexural resistance (kip-ft.)

a = depth of equivalent rectangular stress block (in.)

c = distance from the extreme compression fiber to the neutral axis (in.)

f_{ps} = average stress in prestressing steel at nominal bending resistance (ksi)

ϕ = resistance factor

α_1 = stress block factor, taken as 0.85 for concrete with $f'_c < 10$ ksi

β_1 = stress block factor

ε_t = net tensile strain in extreme tension steel at nominal resistance (in. / in.)

ε_{cl} = compression-controlled strain limit, taken as 0.002 in. / in. (5.6.2.1)

ε_{tl} = tension-controlled strain limit, taken as 0.005 in. / in. (5.6.2.1)

A_{ps} = total area of prestressing steel (in.^{2})

d_p = distance from extreme compression fiber to the centroid of the prestressing tendons (in.)

b = width of the compression face of the member (equals width of beam) (in.)

b_w = web width (equals the total thickness of both side walls of the beam) (in.)

h_f = compression flange depth (equals top slab thickness of the beam) (in.)

f_{pu} = specified tensile strength of prestressing steel (ksi)

f_{pe} = effective stress in the prestressing steel after losses (ksi)

k = 0.28 (Table C5.6.3.1.1-1)

f'_c = specified compressive strength of concrete for use in design (ksi)

d_t = distance from the extreme compression fiber to the centroid of the extreme tension steel element (in.)

Δf_{pT} = total loss (ksi)

The factored flexural resistance equations shown above have been simplified to include only the prestressing steel. All other reinforcement shall be ignored.

Minimum Reinforcement

The Department requires minimum reinforcement be adequate to develop a factored flexural resistance of at least the cracking moment or 1.33 times the Strength I factored moment for prestressed beams. This is done to ensure ductility in the event of an unexpected overload.

$M_r \geq M_{cr}$ (5.6.3.3)

In which:

$$M_{cr} = \gamma_3 \frac{S_b(\gamma_1 f_r + \gamma_2 f_{cpe})}{12} \geq \frac{S_b f_r}{12}$$ (Eq. 5.6.3.3-1)

$$f_r = 0.24\lambda\sqrt{f_c^{'}}$$ (5.4.2.6)

$$f_{cpe} = \frac{F_s}{A} + \frac{F_s e}{S_b}$$

Where:

M_r = factored flexural resistance of a section in bending (kip-ft.)

M_{cr}= cracking moment (kip-ft.)

f_r = modulus of rupture of concrete (ksi)

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

S_b = non-composite section modulus for the bottom fiber of the beam (in.^{3})

$f_c^{'}$ = specified compressive strength of concrete for use in design (ksi)

F_S = total prestressing force after all losses (kips)

A = area of beam (in.^{2})

e = eccentricity of centroid of strand pattern from NA of beam (in.)

γ_1 = flexural cracking variability factor

= 1.6 for non-segmentally constructed members

γ_2 = prestress variability factor

= 1.1 for bonded tendons

γ_3 = ratio of specified minimum yield strength to ultimate tensile strength of reinforcement

= 1.00 for prestressed concrete structures

λ = lightweight concrete factor, taken as 1.0 for regular weight concrete

Calculate Camber and Deflection

Camber, which is the result of the difference between the upward deflection caused by the prestressing forces and the downward deflection due to the weight of the beam and overlay, must be considered when determining the seat elevations. The top of the beam shall be set to provide the minimum overlay thickness specified on the plans.

Camber will vary with the age of the member, primarily because of two factors; loss of prestress which will tend to decrease the deflection, and creep which will tend to increase the deflection. Because of this, correction factors are used in the equations for calculating camber. Factors of 1.80 and 1.85 are used on the upward deflection caused by the prestressing force and downward deflection due to member weight, respectively. These factors are based on the PCI Design Handbook for the time at erection and have been incorporated into the equations shown below.

The deflection due to the transverse tie diaphragms need only be considered for members 60 feet and longer.

Initial Resultant Camber

Camber = $D_{cp} - D_{cb}$

In which:

$$D_{cp} = \frac{F_t(12L)^2 e}{8E_{ci} I}(1.80)$$

$$D_{cb} = \frac{5 w_{net}(12L)^4}{384(12)E_{ci} I}(1.85)$$ (uniform net section weight per foot)

$$D_{cb} = \frac{P_T(12L)^3}{48 E_{ci} I}(1.85)$$ (one transverse tie point load at center)

$$D_{cb} = \frac{P_T L_{tt}(12)}{24 E_{ci} I}\left[3(12\,L)^2 - 4(12\,L_{tt})^2\right](1.85)$$ (two transverse tie point loads symmetrically placed)

Where:

D_{cp} = upward deflection due to prestressing (in.)

D_{cb} = downward deflection due to beam weight (in.)

F_t = total prestressing force immediately after transfer (kips)

L = span length (ft.)

e = eccentricity of centroid of strand pattern from NA of beam (in.)

E_{ci} = modulus of elasticity of concrete at transfer (ksi)

I = moment of inertia of beam (in.^{4})

w_{net} = weight per unit length of the section of the beam with voids (kip/ft.)

P_T = weight of transverse tie diaphragm (kips)

L_{tt} = distance from support to transverse tie (ft.)

Final Resultant Camber for Computing Bearing Seat Elevations

The dead loads to be considered for adjusting the grade line are those which will appreciably increase the downward deflection of the beams after they have been erected. This load is the weight of the initial wearing surface. The weight of future wearing surface is not included.

Normally, the deflection caused by the weight of curbs and rails is insignificant and can be disregarded. In cases where they might appear significant, the above dead loads should be included when adjusting the grade line for dead load deflections.

Camber = $D_{cp} - D_{cb} - D_{ws}$

In which:

$$D_{ws} = \frac{5\,w_{ws}(12L)^4}{384(12)\,E_c\,I}$$

Where:

D_{cp} = upward deflection due to prestressing (in.)

D_{cb} = downward deflection due to beam weight (in.)

D_{ws} = downward deflection due to wearing surface (in.)

w_{ws} = weight per unit length of the wearing surface (kip/ft.)

L = span length (ft.)

E_c = modulus of elasticity of concrete (ksi)

I = moment of inertia of beam (in.^{4})

Downward Deflections Due to Overlay Weight for Adjusting Grade Elevations

@0.25 point = $0.7125D_{ws}$

@0.50 point = D_{ws}

@0.75 point = $0.7125D_{ws}$

Example

60 ft., single span, 27 in. x 36 in. PPC Deck Beam, 33 ft. deck width, 5 in. minimum concrete overlay, Type SM rail, 50 pounds per square foot future wearing surface, 25 degree skew, 2 design lanes, straight grade and HL-93 loading on pile bent abutments.

General Data

Design code	=	LRFD
Span length	=	60 ft.
Beam section	=	27 in. x 36 in. PPC Deck Beam
Roadway width	=	33 ft.
Number of beams	=	11
Overlay thickness	=	5 in. min; 6.0 in. max based on estimated camber; 5.5 in. avg.
Estimated camber	=	1 in.
Type SM rail	=	0.075 k/ft.
FWS	=	50 psf
Relative humidity	=	70 %
Strands	=	½ in. diameter – 270 ksi low relaxation strands
Skew	=	25 degrees

Live Load Data

Loading = HL-93

IM = 1.33 (HL-93); 1.15 (fatigue truck) (3.6.2)

N_L = 2 (3.6.1.1.1)

Trial Strand Pattern

Select strand pattern 20SS from planning charts in BM Section 2.3.6.1.2.

$$A_{ps} = 20\left(0.153\text{ in.}^2\right) = 3.06\text{ in.}^2$$

$$e = 8.35\text{ in.}$$

Materials

Precast Concrete Beam

f'_c = 6.0 ksi

f'_{ci} = 5.0 ksi

f_{pbt} = 201.96 ksi

f_{pu} = 270.0 ksi

Fi/strand = 30.9 kips

Section Properties

Modulus of Elasticity

$E_{ci} = 120{,}000 K_1 w_c^{2.0} f'^{0.33}_{ci}$ (Eq. 5.4.2.4-1)

$E_c = 120{,}000 K_1 w_c^{2.0} f'^{0.33}_{c}$

E_{ci} (beam) = $120{,}000(1.0)(0.146 \text{ k/ft.})^{2.0}(5.0)^{0.33}$ = 4351 ksi

E_c (beam) = $120{,}000(1.0)(0.146 \text{ k/ft.})^{2.0}(6.0)^{0.33}$ = 4620 ksi

E_p (strand) = 28500 ksi (5.4.4.2)

Beam Section

See Table 3.5.4-1 in the BM section 3.5 for deck beam design properties.

A = 569.9 in.^{2}

I = 49697 in.^{4}

K = 0.81

S_b = 3738.1 in.^{3}

S_t = 3626.1 in.^{3}

C_b = 13.30 in.

C_t = 13.71 in.

Distribution Factors *(4.6.2.2)*

<u>Moment</u>

Regardless of the number of loaded lanes the distribution factor equals:

$$g = \frac{S}{D}$$

In which:

$$C = K\left(\frac{W}{L}\right) \leq K$$

$$= 0.81\left(\frac{33\text{ ft.}}{60\text{ ft.}}\right) \leq 0.81$$

$$= 0.45 \leq 0.81$$

$$= 0.45$$

$$D = 11.5 - N_L + 1.4N_L(1 - 0.2C)^2 \quad \text{when } C \leq 5$$

$$= 11.5 - 2 + 1.4(2)[1 - 0.2(0.45)]^2$$

$$= 11.82\text{ ft.}$$

$$g = \frac{3\text{ ft.}}{11.82\text{ ft.}}$$

$$= 0.254$$

<u>Moment (fatigue loading)</u>

$$g_1\text{ (fatigue)} = \frac{g}{m} = \frac{0.254}{1.2} = 0.212$$

Transverse Ties

Number ties $= \left(\frac{L}{25} - 1\right) \geq 1$ rounded up to the nearest integer

$= \left(\frac{60\text{ ft.}}{25} - 1\right) \geq 1$

$= 1.4 \geq 1$ Therefore use 2 @ 1/3 points (1 @ 20 ft.; 1 @ 40 ft.)

Dead Loads

Calculate dead loads in terms of weight per foot per beam.

DC1:

Beam (net)		=	0.594 k/ft.	(BM Table 3.5.4-1)
Beam (solid)		=	0.986 k/ft.	(BM Table 3.5.4-1)
Railing	$\frac{(0.075\text{ k/ft.})}{3\text{ beams}}$	=	0.025 k/ft.	
Shear Key		=	0.025 k/ft.	(BM Table 3.5.4-1)

DW1:

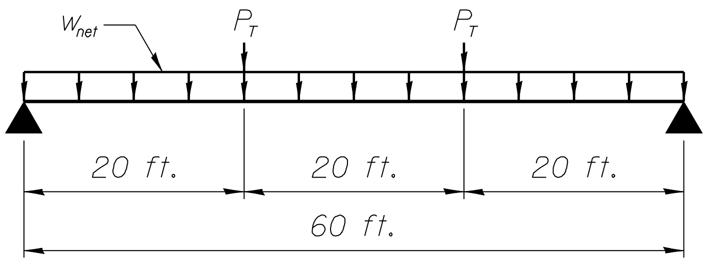
Wearing Surface $(0.15\text{ k/ft.}^3)\left(\frac{5.5\text{ in}}{12\text{ in./ft.}}\right)(3.0\text{ ft.})$ = 0.206 k/ft.

FWS $(0.050\text{ k/ft.}^2)(3.0\text{ ft.})$ = 0.150 k/ft.

Maximum Unfactored Distributed Moments

DC1:

Beam:



$$M_b = \frac{w_{net}L^2}{8} + P_T L_T \qquad \text{(formula for 2 symmetrically placed ties)}$$

In which:

$$P_T = \frac{2\text{ ft.}}{\cos(\text{skew})}(w_{solid} - w_{net})$$

$$= \frac{2\text{ ft.}}{\cos 25^\circ}(0.986\text{ k/ft.} - 0.594\text{ k/ft.})$$

$$= 0.9\text{ kips}$$

$$M_b = \frac{0.594\text{ k/ft.}(60\text{ ft.})^2}{8} + 0.9\text{ kips}(20\text{ ft.})$$

$$= 285.3\text{ k-ft.}$$

Railing and shear key:

$$M = \frac{(w_r + w_{sk})L^2}{8}$$

$$= \frac{(0.025\text{ k/ft.} + 0.025\text{ k/ft.})(60\text{ ft.})^2}{8}$$

$$= 22.5\text{ k-ft.}$$

Total M_{DC1} = 285.3 k-ft. + 22.5 k-ft.

= 307.8 k-ft.

DW1:

Wearing surface and FWS:

$$M = \frac{(w_{ws} + w_{fws})L^2}{8}$$

$$= \frac{(0.206 \text{ k/ft.} + 0.150 \text{ k/ft.})(60 \text{ ft.})^2}{8}$$

$$= 160.2 \text{ k-ft.}$$

Total M_{DW1} = 160.2 k-ft.

LL+IM:

The maximum overall moment occurs at 0.5L for this example. However the maximum live load moment from the LRFD moment tables in the BM Appendix Section 4 may conservatively be used since it's typically slightly higher than the live load at 0.5L.

$$M_{LL+IM} = g(M_{LL+IM\ (undistributed)})$$

Where:

$M_{LL+IM\ (undistributed)}$ = 1352.0 k-ft. (from computer software)

$$M_{LL+IM} = 0.254(1352.0)$$

$$= 343.4 \text{ k-ft.}$$

FL+IM:

$$M_{FL+IM} = g(M_{FL+IM\ (undistributed)})$$

Where:

$M_{FL+IM\ (undistributed)}$ = 625.6 k-ft. (from computer software)

$$M_{FL+IM} = 0.212(625.6)$$

$$= 132.6 \text{ k-ft.}$$

Prestress Losses *(5.9.3)*

Total Loss of Prestress

$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$ (Eq. 5.9.3.1-1)

Instantaneous Losses (due to elastic shortening):

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ (Eq. 5.9.3.2.3a-1)

Assume F_t equals 90 percent of F_i for first iteration:

$$F_i = A_{ps}(f_{pbt}) = 3.06\text{ in.}^2(201.96\text{ ksi}) = 618\text{ kips}$$

$$F_t = 0.9(F_i) = 0.9(618\text{ kips}) = 556\text{ kips}$$

$$M_b = 285.3\text{ k-ft.}$$

$$f_{cgp} = \frac{F_t}{A} + \frac{F_t e^2}{I} - \frac{M_b(12)e}{I}$$

$$= \frac{556\text{ kips}}{569.9\text{ in.}^2} + \frac{(556\text{ kips})(8.35\text{ in.})^2}{49697\text{ in.}^4} - \frac{(285.3\text{ k}-\text{ft.})(12\text{ in./ft.})(8.35\text{ in.})}{49697\text{ in.}^4}$$

$$= 1.18\text{ ksi}$$

Calculate Δf_{pES}:

$$\Delta f_{pES} = \frac{28500\text{ ksi}}{4351\text{ ksi}}(1.18\text{ ksi}) = 7.73\text{ ksi}$$

Check Assumption:

$$\frac{(f_{pbt} - \Delta f_{pES})}{f_{pbt}} = \frac{(201.96\text{ ksi} - 7.73\text{ ksi})}{201.96\text{ ksi}} = 0.96 > 0.90$$

Recalculate

Assume F_t equals 96 percent of F_i for second iteration:

$$F_t = 0.96(618\text{ kips}) = 593\text{ kips}$$

$$f_{cgp} = \frac{593\text{ kips}}{569.9\text{ in.}^2} + \frac{(593\text{ kips})(8.35\text{ in.})^2}{49697\text{ in.}^4} - \frac{(285.3\text{ k}-\text{ft.})(12\text{ in./ft.})(8.35\text{ in.})}{49697\text{ in.}^4}$$

$= \quad 1.30 \text{ ksi}$

Calculate Δf_{pES}:

$$\Delta f_{pES} = \frac{28500 \text{ ksi}}{4351 \text{ ksi}}(1.30 \text{ ksi}) = 8.52 \text{ ksi}$$

Check Assumption:

$$\frac{(f_{pbt} - \Delta f_{pES})}{f_{pbt}} = \frac{(201.96 \text{ ksi} - 8.52 \text{ ksi})}{201.96 \text{ ksi}} = 0.96 \quad \text{Ok}$$

Time Dependent Losses:

$$\Delta f_{pLT} = 10.0\frac{f_{pbt}A_{ps}}{A}\gamma_h\gamma_{st} + 12.0\gamma_h\gamma_{st} + \Delta f_{pR} \qquad \text{(Eq. 5.9.3.3-1)}$$

In which:

$$\gamma_h = 1.7 - 0.01H = 1.7 - 0.01(70) = 1.0 \qquad \text{(Eq. 5.9.3.3-2)}$$

$$\gamma_{st} = \frac{5}{(1 + f'_{ci})} = \frac{5}{(1 + 5 \text{ ksi})} = 0.833 \qquad \text{(Eq. 5.9.3.3-3)}$$

$\Delta f_{pR} = 2.4 \text{ ksi}$

$$\Delta f_{pLT} = 10.0\frac{(201.96 \text{ ksi})(3.06 \text{ in.}^2)}{569.9 \text{ in.}^2}(1.0)(0.833) + 12.0(1.0)(0.833) + 2.4 \text{ ksi} = 21.43 \text{ ksi}$$

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} = 8.52 \text{ ksi} + 21.43 \text{ ksi} = 29.95 \text{ ksi}$$

$$\%\text{Loss} = \frac{29.95 \text{ ksi}}{201.96 \text{ ksi}} = 14.8\ \%$$

Stress Limits for Concrete

Temporary stresses (5.9.4.1)

Compression:

$$0.65f'_{ci} = 0.65(5.0\text{ ksi}) = 3.25\text{ ksi}$$

Tension:

$$0.24\lambda\sqrt{f'_{ci}} = 0.24(1.0)\sqrt{5.0\text{ ksi}} = 0.537\text{ ksi}$$

Service stresses after losses (5.9.4.2)

Compression (For Service I load combination):

$$0.60\phi_w f'_c = 0.60(1.0)(6.0\text{ ksi}) = 3.6\text{ ksi} \quad \text{(a)}$$

$$0.45f'_c = 0.45(6.0\text{ ksi}) = 2.7\text{ ksi} \quad \text{(b)}$$

Tension (For Service III load combination):

$$0.19\lambda\sqrt{f'_c} = 0.19(1.0)\sqrt{6.0\text{ ksi}} = 0.465\text{ ksi}$$

Fatigue stresses after losses (5.5.3.1)

Compression (For Fatigue I load combination):

$$0.40f'_c = 0.40(6.0\text{ ksi}) = 2.40\text{ ksi}$$

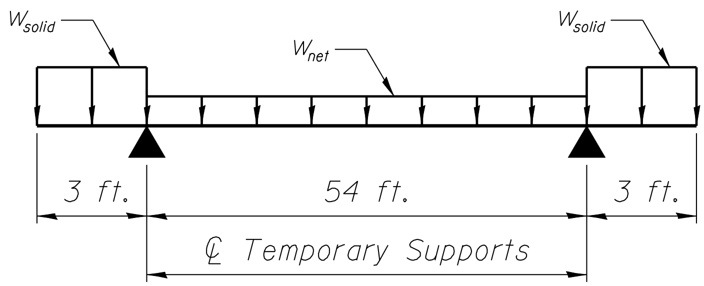
Tension limit for determination of cracking of section:

$$\text{Uncracked} \le 0.095\sqrt{f'_c} \le \text{Cracked}$$

$$0.095\sqrt{6.0\text{ ksi}} = 0.233\text{ ksi}$$

Check Temporary Stresses

Calculate Dead Load Moments for Determining Temporary Stresses



@ Temporary Supports:

$$M_{bts} = \frac{w_{solid}(3\text{ ft.})^2}{2} = \frac{(0.986\text{ k/ft.})(3\text{ ft.})^2}{2} = 4.4\text{ k-ft.}$$

@ Center:

$$M_{bts} = \frac{w_{net}(L_{cts})^2}{8} - \frac{w_{solid}(3\text{ ft.})^2}{2}$$

$$= \frac{(0.594\text{ k/ft.})(54.0\text{ ft.})^2}{8} - \frac{(0.986\text{ k/ft.})(3\text{ ft.})^2}{2} = 212.1\text{ k-ft.}$$

Prestress Force Immediately after Transfer

$$F_t = A_{ps}(f_{pbt} - \Delta f_{pES}) = (3.06\text{ in.}^2)(201.96\text{ ksi} - 8.52\text{ ksi}) = 592\text{ kips}$$

Temporary Stresses

@ Temporary supports:

$$f_t = \frac{F_t}{A} - \frac{F_t e}{S_t} - \frac{M_{bts}(12)}{S_t}$$

$$= \frac{592\text{ kips}}{569.9\text{ in.}^2} - \frac{(592\text{ kips})(8.35\text{ in.})}{3626.1\text{ in.}^3} - \frac{(4.4\text{ k-ft.})(12\text{ in./ft.})}{3626.1\text{ in.}^3}$$

$$= 0.339\text{ ksi (tension)} \leq 0.537\text{ ksi} \quad \text{Ok}$$

$$f_b = \frac{F_t}{A} + \frac{F_t e}{S_b} + \frac{M_{bts}(12)}{S_b}$$

$$= \frac{592\text{ kips}}{569.9\text{ in.}^2} + \frac{(592\text{ kips})(8.35\text{ in.})}{3738.1\text{ in.}^3} + \frac{(4.4\text{ k}-\text{ft.})(12\text{ in./ft.})}{3738.1\text{ in.}^3}$$

= 2.375 ksi (comp.) ≤ 3.000 ksi Ok

@ Center:

$$f_t = \frac{F_t}{A} - \frac{F_t e}{S_t} + \frac{M_{bts}(12)}{S_t}$$

$$= \frac{592\text{ kips}}{569.9\text{ in.}^2} - \frac{(592\text{ kips})(8.35\text{ in.})}{3626.1\text{ in.}^3} + \frac{(212.1\text{ k}-\text{ft.})(12\text{ in./ft.})}{3626.1\text{ in.}^3}$$

= 0.377 ksi (comp.) ≤ 3.000 ksi Ok

$$f_b = \frac{F_t}{A} + \frac{F_t e}{S_b} - \frac{M_{bts}(12)}{S_b}$$

$$= \frac{592\text{ kips}}{569.9\text{ in.}^2} + \frac{(592\text{ kips})(8.35\text{ in.})}{3738.1\text{ in.}^3} - \frac{(212.1\text{ k}-\text{ft.})(12\text{ in./ft.})}{3738.1\text{ in.}^3}$$

= 1.680 ksi (comp.) ≤ 3.000 ksi Ok

Check Service Stresses after Losses

Prestress Force after Losses:

$$F_s = A_{ps}(f_{pbt} - \Delta f_{pT}) = (3.06 \text{ in.}^2)(201.96 \text{ ksi} - 29.95 \text{ ksi}) = 526 \text{ kips}$$

Service Stresses:

@ Center:

$$f_t = \frac{F_S}{A} - \frac{F_S e}{S_t} + \frac{(M_{DC1} + M_{DW1} + M_{LL+IM})(12)}{S_t} \quad \text{(a)}$$

$$= \frac{526 \text{ kips}}{569.9 \text{ in.}^2} - \frac{(526 \text{ kips})(8.35 \text{ in.})}{3626.1 \text{ in.}^3}$$

$$+ \frac{(307.8 \text{ k-ft.} + 160.2 \text{ k-ft.} + 343.4 \text{ k-ft.})(12 \text{ in./ft.})}{3626.1 \text{ in.}^3}$$

$= 2.397$ ksi (comp.) $\leq$ 3.600 ksi Ok

$$f_t = \frac{F_S}{A} - \frac{F_S e}{S_t} + \frac{(M_{DC1} + M_{DW1})(12)}{S_t} \quad \text{(b)}$$

$$= \frac{526 \text{ kips}}{569.9 \text{ in.}^2} - \frac{(526 \text{ kips})(8.35 \text{ in.})}{3626.1 \text{ in.}^3} + \frac{(307.8 \text{ k-ft.} + 160.2 \text{ k-ft.})(12 \text{ in./ft.})}{3626.1 \text{ in.}^3}$$

$= 1.260$ ksi (comp.) $\leq$ 2.700 ksi Ok

$$f_b = \frac{F_s}{A} + \frac{F_s e}{S_b} - \frac{(M_{DC1} + M_{DW1})(12)}{S_b} - (0.8)\frac{M_{LL+IM}(12)}{S_b}$$

$$= \frac{526 \text{ kips}}{569.9 \text{ in.}^2} + \frac{(526 \text{ kips})(8.35 \text{ in.})}{3738.1 \text{ in.}^3} - \frac{(307.8 \text{ k-ft.} + 160.2 \text{ k-ft.})(12 \text{ in./ft.})}{3738.1 \text{ in.}^3}$$

$$- (0.8)\frac{(343.4 \text{ k-ft.})(12 \text{ in./ft.})}{3738.1 \text{ in.}^3}$$

$= 0.286$ ksi (tension) $\leq$ 0.465 ksi Ok

Check Fatigue Stresses after Losses

Determine if section is cracked for fatigue investigions:

@ Center:

$$f_b = \frac{F_s}{A} + \frac{F_s e}{S_b} - \frac{(M_{DC1} + M_{DW1})(12)}{S_b} - (1.5)\frac{M_{FL+IM}(12)}{S_b}$$

$$= \frac{526 \text{ kips}}{569.9 \text{ in.}^2} + \frac{(526 \text{ kips})(8.35 \text{ in.})}{3738.1 \text{ in.}^3} - \frac{(307.8 \text{ k-ft.} + 160.2 \text{ k-ft.})(12 \text{ in./ft.})}{3738.1 \text{ in.}^3}$$

$$- (1.5)\frac{(132.6 \text{ k-ft.})(12 \text{ in./ft.})}{3738.1 \text{ in.}^3}$$

$= 0.043$ ksi (tension) $\leq$ 0.233 ksi Use uncracked section properties

Fatigue Stresses:

@ Center:

$$f_t = 0.5\left[\frac{F_S}{A} - \frac{F_S e}{S_t} + \frac{(M_{DC1} + M_{DW1})(12)}{S_t}\right] + 1.5\frac{M_{FL+IM}(12)}{S_t}$$

$$= 0.5\left(\frac{526 \text{ kips}}{569.9 \text{ in.}^2}\right) - 0.5\frac{(526 \text{ kips})(8.35 \text{ in.})}{3626.1 \text{ in.}^3}$$

$$+ 0.5\frac{(307.8 \text{ k-ft.} + 160.2 \text{ k-ft.})(12 \text{ in./ft.})}{3626.1 \text{ in.}^3} + 1.5\frac{(132.6 \text{ k-ft.})(12 \text{ in./ft.})}{3626.1 \text{ in.}^3}$$

$= 1.268$ ksi (comp.) $\leq$ 2.400 ksi Ok

Check Factored Flexural Resistance

Strength I Moment:

$M_u = 1.25(M_{DC1}) + 1.5(M_{DW1}) + 1.75(M_{LL+IM})$

$= 1.25(307.8 \text{ k-ft.}) + 1.5(160.2 \text{ k-ft.}) + 1.75(343.4 \text{ k-ft.})$

$= 1226.0$ k-ft.

Factored Flexural Resistance:

$M_r = \phi M_n$

Calculate Compression Block Depth (assume rectangular):

$$c = \frac{A_{ps} f_{pu}}{\alpha_1 f'_c \beta_1 b + k A_{ps} \frac{f_{pu}}{d_p}}$$ (Eq. 5.6.3.1.1-4)

In which:

f'_c = beam concrete strength = 6.0 ksi

$\beta_1 = 0.65 \le 0.85 - 0.05(f'_c - 4.0) \le 0.85$ (5.6.2.2)

$= 0.65 \le 0.85 - 0.05(6.0 \text{ ksi} - 4.0) \le 0.85$

$= 0.65 \le 0.75 \le 0.85$

$= 0.75$

$\alpha_1 = 0.85$ (5.6.2.2)

b = beam width = 36 in.

k = 0.28

$d_p = C_t + e$

= 13.71 in. + 8.35 in.

= 22.06 in.

$$c = \frac{(3.06 \text{ in.}^2)(270 \text{ ksi})}{0.85(6.0 \text{ ksi})(0.75)(36 \text{ in.}) + 0.28(3.06 \text{ in.}^2)\frac{270 \text{ ksi}}{22.06 \text{ in.}}}$$

= 5.58 in.

$a = \beta_1 c$

= 0.75(5.58 in.)

= 4.19 in. ≤ 5.5 in. Therefore Rectangular Section

Calculate Nominal Flexural Resistance:

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)\left(\frac{1}{12}\right)$$ (Eq. 5.6.3.2.2-1)

In which:

Check $f_{pe} \ge 0.5 f_{pu}$ to verify use of Eq. 5.7.3.1.1-1 (5.6.3.1.1)

$f_{pe} = f_{pu} - \Delta f_{pT}$

$= 270\text{ ksi} - 30.07\text{ ksi}$

$= 239.93\text{ ksi} \geq 0.5(270\text{ ksi}) = 135\text{ ksi}$ Ok

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) \quad \text{(Eq. 5.6.3.1.1-1)}$$

$$= 270\text{ ksi}\left(1 - 0.28\frac{5.58\text{ in.}}{22.06\text{ in.}}\right)$$

$$= 251\text{ ksi}$$

$$M_n = \left[\left(3.06\text{ in.}^2\right)\left(251\text{ ksi}\right)\left(22.06\text{ in.} - \frac{4.19\text{ in.}}{2}\right)\right]\left(\frac{1}{12\text{ in./ft.}}\right)$$

$$= 1278\text{ k-ft.}$$

Calculate ϕ:

$$\phi = 0.75 \leq 0.75 + \frac{0.25(\varepsilon_t - \varepsilon_{cl})}{(\varepsilon_{tl} - \varepsilon_{cl})} \leq 1.0 \quad \text{(Eq. 5.5.4.2.1-1)}$$

In which:

$$\varepsilon_t = \frac{0.003(d_t - c)}{c} \quad \text{(C5.6.2.1)}$$

d_t = depth of beam – distance from bottom of beam to bottom row of strands

= 27 in. – 1.75 in.

= 25.25 in.

c = 5.58 in.

$$\varepsilon_t = \frac{0.003(25.25\text{ in.} - 5.58\text{ in.})}{5.58\text{ in.}}$$

$= 0.011$

$\varepsilon_{cl} = 0.002$

$\varepsilon_{tl} = 0.005$

$$\phi = 0.75 \leq 0.75 + \frac{0.25(0.011 - 0.002)}{(0.005 - 0.002)} \leq 1.0$$

$$= 0.75 \leq 1.42 \leq 1.0$$

$= 1.0$

$M_r = 1.0(1278\text{ k-ft.})$

$= 1278\text{ k-ft.} \geq 1226.0\text{ k-ft.}$ Ok

Check Minimum Prestressing Steel:

$M_r \geq M_{cr}$ (5.6.3.3.2)

In which:

$f_r = 0.24\sqrt{f'_c}$ (5.4.2.6)

$= 0.24\sqrt{6.0\text{ ksi}}$

$= 0.59\text{ ksi}$

$$f_{cpe} = \frac{F_s}{A} + \frac{F_s e}{S_b}$$

$$= \frac{526\text{ kips}}{569.9\text{ in.}^2} + \frac{(526\text{ kips})(8.35\text{ in.})}{3738.1\text{ in.}^3}$$

$= 2.10\text{ ksi}$

$$M_{cr} = \gamma_3 \frac{S_b(\gamma_1 f_r + \gamma_2 f_{cpe})}{12} \geq \frac{S_b f_r}{12} \quad \text{(Eq. 5.6.3.3.2-1)}$$

$$= 1.00\frac{(3738.1\text{ in.}^3)(1.6(0.91\text{ ksi}) + 1.1(2.10\text{ ksi}))}{(12\text{ in./ft.})} \geq \frac{3738.1\text{ in.}^3(0.49\text{ ksi})}{(12\text{ in./ft.})}$$

$= 1173\text{ k-ft.} \geq 153\text{ k-ft.}$

$= 1173\text{ k-ft.}$

$M_r \geq 1173\text{ k-ft.}$

$1278\text{ k-ft.} \geq 1173\text{ k-ft.}$ Ok

Calculate Camber and Deflection

Initial Resultant Camber

Camber = D_{cp} - D_{cb}

In which:

$$D_{cp} = \frac{F_t (12L)^2 e}{8 E_{ci} I}(1.80)$$

$$= \frac{(592 \text{ kips})[(12 \text{ in./ft.})(60 \text{ ft.})]^2 (8.35)}{8(4287 \text{ ksi})(49697 \text{ in.}^4)}(1.80)$$

= 2.71 in. up

$$D_{cb} = \frac{5 w_{net} (12L)^4}{384 (12) E_{ci} I}(1.85) \qquad \text{(net beam section)}$$

$$= \frac{5(0.594 \text{ k/ft.})[(12 \text{ in./ft.})(60 \text{ ft.})]^4}{384 (12 \text{ in./ft.})(4287 \text{ ksi})(49697 \text{ in.}^4)}(1.85)$$

= 1.50 in. down

$$D_{cb} = \frac{P_T L_{tt} (12)}{24 E_{ci} I}\left[3(12 L)^2 - 4(12 L_{tt})^2\right](1.85) \qquad \text{(tie diaphragms)}$$

$$= \frac{0.9 \text{ kips}(20 \text{ ft.})(12 \text{ in./ft.})}{24(4287 \text{ ksi})(49697 \text{ in.}^4)}\left[3(12 \text{ in./ft.})^2 (60 \text{ ft.})^2 - 4(12 \text{ in./ft.})^2 (20 \text{ ft.})^2\right](1.85)$$

= 0.10 in. down

Camber = 2.71 in.-1.50 in.-0.10 in.

= 1.11 in. up

Final Resultant Camber for Computing Bearing Seat Elevations

Camber = D_{cp} - D_{cb} - D_{ws}

In which:

$$D_{ws} = \frac{5 w_{ws} (12L)^4}{384 (12) E_c I}$$

$$= \frac{5(0.206\text{ k/ft.})[(12\text{ in./ft.})(60\text{ ft})]^4}{384(12\text{ in./ft.})(4696\text{ ksi})(49697\text{ in.}^4)}$$

= 0.26 in. down

Camber = 2.71 in.-1.50 in.-0.10 in.-0.26 in.

= 0.85 in. up

0.85 in. is close enough to the 1 in. assumed camber therefore no reiteration is necessary.

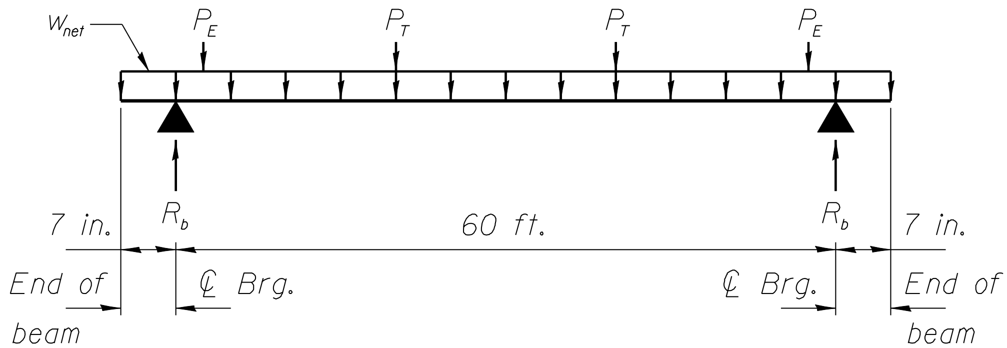
<u>Downward Deflections Due to Overlay Weight for Adjusting Grade Elevations</u>

@0.25 point = $0.7125D_{ws}$ = 0.7125(0.26 in.) = 0.19 in.

@0.50 point = D_{ws} = 0.26 in.

@0.75 point = $0.7125D_{ws}$ = 0.7125(0.26 in.) = 0.19 in.

Beam Weight Calculation for Design of Lifting Loops and Substructure



Calculate Weight of Transverse Tie Diaphragm (P_T)

$$P_T = \frac{2\text{ ft.}}{\cos(\text{skew})}\left(w_{solid} - w_{net}\right)$$

$$= \frac{2\text{ ft.}}{\cos 25^\circ}\left(0.986\text{ k/ft.} - 0.594\text{ k/ft.}\right)$$

$$= 0.9\text{ kips}$$

Calculate Weight of End Block (P_E)

$$P_E = L_E\left(w_{solid} - w_{net}\right)$$ (See BM Fig. 3.5.9-3 for calculation of L_E)

In which:

$$L_E = B + W\tan(\text{skew})$$

In which:

$$B = \text{larger of } \frac{A}{\cos(\text{skew})} - W\tan(\text{skew}) \quad \text{or} \quad 27 + V\tan(\text{skew})$$

Where:

A = 30 in.

W = 6 in.

V = 24 in.

$$B = \frac{30\text{ in.}}{\cos 25^\circ} - 6\text{ in.}\left(\tan 25^\circ\right) \quad \text{or} \quad 27\text{ in.} + 24\text{ in.}\left(\tan 25^\circ\right)$$

B = 30.3 in. or 38.2 in. use 38.2 in.

$$L_E = 38.2 \text{ in.} + 6 \text{ in.} \left(\tan 25^{o}\right) = 41.0 \text{ in.}$$

$$P_E = 41.0 \text{ in.}(1 \text{ ft.}/12 \text{ in.})(0.986 \text{ k/ft.} - 0.594 \text{ k/ft.})$$

$$= 1.3 \text{ kips}$$

$$\text{Total Beam Weight} = \left[0.594 \text{ k/ft.}(61.17 \text{ ft.}) + 2(1.3 \text{ k}) + 2(0.9 \text{ k})\right]\left(\frac{1000 \text{ lb.}}{\text{k}}\right)$$

$= 40735$ lbs. $\therefore$ Use 2 lifting loops at each end (See BM Fig. 3.5.9-4)

$$R_b = (0.5)(40735 \text{ lbs.})\left(\frac{1 \text{ k}}{1000 \text{ lbs.}}\right) = 20.4 \text{ k/beam}$$