LRFD Culvert Flexure Design

The requirements for the design of concrete box culverts are found in Chapter 12 of the AASHTO LRFD Bridge Design Specifications. Article 12.11.1 of Section 12: Buried Structures and Tunnel Liners states that the applicable provisions of the specifications shall be used except as provided otherwise. The procedure for determining the applied forces and calculating the appropriate factored loads can be found in the current version of the Culvert Manual.

Top slabs and sidewalls of cast in place reinforced concrete culverts shall be investigated for combined bending and uniaxial axial load and the effects of slenderness must be taken into account. A common detailing practice of determining the minimal sidewall thickness equal to 1 in. per foot of clear height of culvert may create a condition of having a slender member and the moment magnification provisions of Article 4.5.3.2.2 would then be applicable.

For flexural design of members in concrete box culverts, the designer must check the strength and service limit states. Minimum reinforcement and bar spacing requirements also apply. According to Article 5.5.3.1, fatigue need not be investigated for reinforced concrete box culverts.

LRFD Culvert Flexure Design, Procedure, and Outline

Concrete members should be proportioned using reinforcement equal to $0.375\rho_{bal}$ or less. This will ensure that the strain in the steel reinforcement will always exceed 0.005, making it such that ϕ can always be taken as 0.9. For low-fill cases where a thin slab with heavy reinforcement is required, this can be exceeded, but the maximum amount of reinforcement should never exceed $0.60\rho_{bal}$, despite this technically being allowed by the code. This effectively re-introduces the maximum reinforcement provision concepts that existed in the AASHTO LRFD Bridge Design Specifications prior to 2005 and in the AASHTO Standard Specifications for Highway Bridges, 17th Ed.

The balanced reinforcement ratio may be calculated according to Eq. 8-18 of Article 8.16.3.2 from the AASHTO Standard Specifications for Highway Bridges, 17th Ed.

$$\rho_{bal} = \frac{0.85\beta_1 f_c'}{f_y}\left(\frac{87{,}000}{87{,}000+f_y}\right)$$

Eq. 8-18, AASHTO Standard Specifications for Highway Bridges, 17th Ed.

Where f_c' and f_y are in units of psi.

For $f_c' = 3500$ psi and $f_y = 60{,}000$ psi, $\rho_{bal} = 0.02494$ and the maximum reinforcement allowed in a concrete member is $0.60\rho_{bal} = 0.01496$ with the preferred amount of reinforcement equal to $0.375\rho_{bal} = 0.00935$.

The following is a list of required design checks for culverts, with their respective article numbers in the AASHTO LRFD Bridge Design Specifications:

Slenderness and Moment Magnification	(5.6.4.3, 4.5.3.2.2)
Flexural Resistance (Axial-Moment Interaction)	(5.6.3.2)
Control of Cracking by Distribution of Reinforcement	(5.6.7)
Minimum Reinforcement	(5.6.3.3)
Reinforcement Spacing Requirements	(5.10.3)
Distribution Reinforcement (top slabs with 2 feet of fill or less only)	(5.12.2)
Shrinkage and Temperature	(5.10.6)

Check Slenderness and Moment Magnification (5.6.4.3, 4.5.3.2.2)

Culvert slabs and walls often times are slender members. When this is the case, the moments must be magnified to account for secondary effects.

Check Slenderness (5.6.4.3)

The provisions of Article 5.6.4.3 determine if slenderness effects must be considered. Culvert members are considered to be braced against sidesway. Therefore, slenderness may be neglected if $\frac{Kl_u}{r} < 34 - 12\frac{M_1}{M_2}$. Given that the connection of the top slab to the sidewall is considered pinned, M_1 is always equal to zero for sidewalls and end spans of multiple barrel culverts. For these cases, the equation simplifies to $\frac{Kl_u}{r} < 34$. For interior spans of multiple barrel culverts, taking $M_1 = M_2$ causes the equation to simplify to $\frac{Kl_u}{r} < 22$.

Therefore check slenderness using:

$\frac{Kl_u}{r} <$ 34 (sidewalls and exterior span slabs) or

$\frac{Kl_u}{r} <$ 22 (interior span slabs)

Where:

K = 1.0 for members braced against sidesway

l_u = clear distance between supports (in.)

r = radius of gyration of member (in.)

= $\frac{t\sqrt{3}}{6}$ for rectangular members, in which t is the member thickness (in.)

If this is satisfied, the moments are used as-is and do not need to be amplified to account for slenderness effects. If not, the moments require magnification. If $\frac{Kl_u}{r} < 100$, the provisions of Article 4.5.3.2.2b may be used.

<u>Calculate Moment Magnifier</u> <u>(4.5.3.2.2)</u>

The maximum factored axial force is used to calculate the moment magnifier which will produce the greatest magnifier to be used to calculate M_c. The magnifier calculated for the condition of maximum axial force may be conservatively applied to the forces from any loading condition.

Because there is no sidesway in the box, the sidesway portion of Equation 4.5.3.2.2b-1 is zero and the equation simplifies to:

$M_c = \delta_b M_{2b}$ (Eq. 4.5.3.2.2b-1)

Where:

M_{2b} = Strength I factored maximum moment

$$\delta_b = \frac{C_m}{1-\frac{P_u}{\phi_K P_e}} \geq 1.0 \qquad \text{(Eq. 4.5.3.2.2b-3)}$$

In which:

C_m = 1.0

P_u = Strength I factored maximum axial load

$$P_e = \frac{\pi^2 EI}{(K\ell_u)^2} \qquad \text{(Eq. 4.5.3.2.2b-5)}$$

ϕ_K = stiffness reduction factor; 0.75 for concrete members

K = 1.0

$$EI = \frac{E_c I_g / 2.5}{1+\beta_d} \qquad \text{(Eq. 5.6.4.3-2)}$$

E_c = $120000K_1 w_c^2 f'_c{}^{0.33}$ (Eq. 5.4.2.4-1)

$$I_g = \frac{bh^3}{12}$$

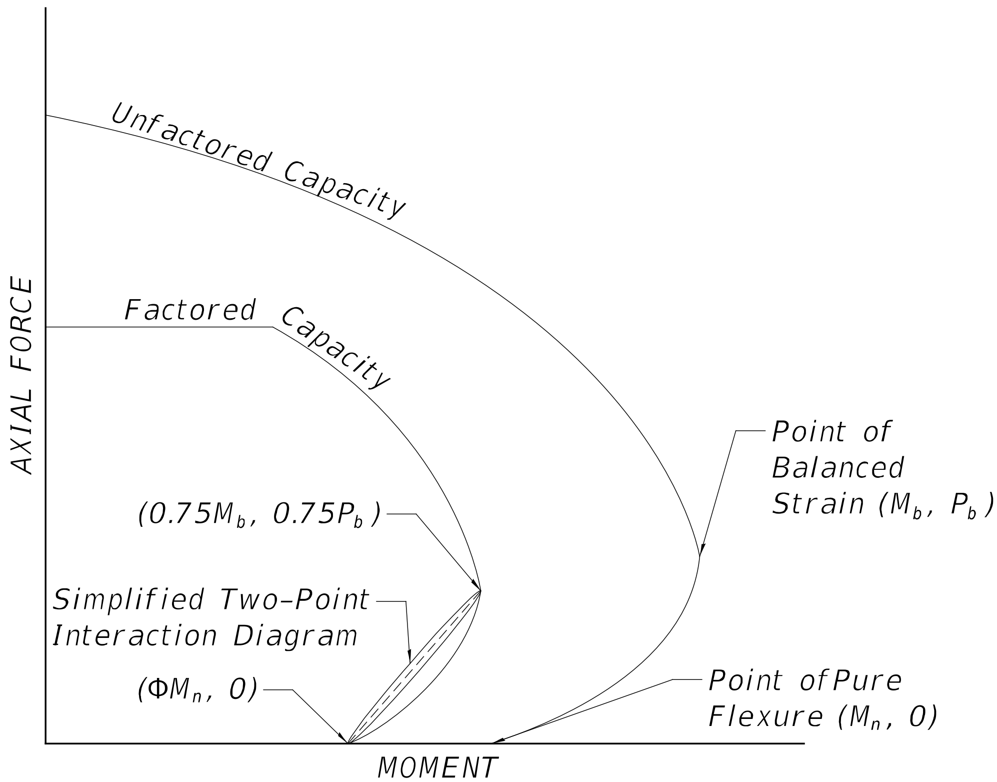
β_d = ratio of maximum factored permanent load moment to maximum factored total load moment, always positive

This value of δ_b will be applied to the moments for all limit states.

Check Flexural Resistance (Axial/Moment Interaction) *(5.6.3.2)*

The effects of compression reinforcement should be neglected to remain consistent with AASHTOWare Bridge Rating.

A combined bending and axial load interaction diagram is required. The members of culverts with fills less than 25 feet are primarily flexural members with a relatively small axial load. Therefore, a diagram suitable for design purposes can be approximated by drawing a line between two points; pure flexure and the balance point. The factored loads are then plotted and checked to verify that they are within the limits of the diagram.



Calculate Point of Pure Flexure

The factored resistance, M_r (k-in.), shall be taken as:

$$M_r = \phi M_n = \phi\left[A_s f_s\left(d_s - \frac{a}{2}\right)\right] \geq M_{STRENGTH1} \quad \text{(Eqs. 5.6.3.2.1-1 \& 5.6.3.2.2-1)}$$

Where:

ϕ = Assumed to be 0.9, then checked using the procedure found in Article 5.5.4.2. In this procedure, the reinforcement strain, ε_t, is calculated, and ϕ is dependent upon this strain. ε_t is calculated assuming similar triangles and a concrete strain of 0.003.

$$\varepsilon_t = \frac{0.003(d_t - c)}{c} \quad \text{(C5.6.2.1)}$$

- If $\varepsilon_t < 0.002$, $\phi = 0.75$

- If $0.002 < \varepsilon_t < 0.005$, $\phi = 0.75 + \dfrac{0.15(\varepsilon_t - \varepsilon_{cl})}{(\varepsilon_{tl} - \varepsilon_{cl})}$
- If $\varepsilon_t > 0.005$, $\phi = 0.9$

Where ε_{cl} is taken as 0.002 and ε_{cl} is taken as 0.005, as stated in Article 5.6.2.1 and Table C5.6.2.1.

a = depth of equivalent stress block (in.), taken as $a = c\beta_1$

c = $\dfrac{A_s f_s}{\alpha_1 \beta_1 f'_c b}$ (in.) (Eq. 5.6.3.1.1-4)

A_s = area of tension reinforcement in strip (in.^{2})

b = width of design strip (in.)

d_s = distance from extreme compression fiber to centroid of tensile reinforcement (in.)

f_s = stress in the mild steel tension reinforcement as specified at nominal flexural resistance (ksi). As specified in Article 5.6.2.1, if $c / d_s < 0.003 / (0.003 + \varepsilon_{cl})$, then f_y may used in lieu of exact computation of f_s. For 60 ksi reinforcement, ε_{cl} is taken as 0.002, making the ratio $0.003 / (0.003 + \varepsilon_{cl})$ equal to 0.6. Typically in design, f_s is assumed to be equal to f_y, then the assumption is checked.

f'_c = specified compressive strength of concrete (ksi)

α_1 = 0.85 for concrete with strength less than 10 ksi (5.6.2.2)

β_1 = stress block factor specified in Article 5.6.2.2

$$\therefore \quad M_r = \phi M_n = \phi\left[A_s f_s\left(d_s - \frac{1}{2}\frac{A_s f_s}{0.85 f'_c b}\right)\right]$$

Calculate Point of Balanced Strain Failure (Balance Point)

The unfactored moment (M_b) and axial force (P_b) that comprise the balance point are calculated using the following equations. These equations are based on the geometry of a rectangular member, and can be found in reinforced concrete design textbooks. They also are found in 8.16.4.2.3 of the AASHTO Standard Specifications for Highway Bridges (17th Edition), albeit with different nomenclature.

$$M_b = 0.85f'_c\,\beta_1 x_b b\left(\frac{h}{2}-\frac{\beta_1 x_b}{2}\right)+A_s f_s\left(d_s-\frac{h}{2}\right)$$

$$P_b = 0.85f'_c\beta_1 x_b b - A_s f_s$$

Where:

f'_c = specified compressive strength of concrete (ksi)

β_1 = stress block factor specified in Article 5.6.2.2

$x_b = \dfrac{87d_s}{f_y + 87}$

b = 12 in.

h = member thickness (in.)

A_s = area of tension reinforcement in strip (in.^{2})

f_s = stress in the mild steel tension reinforcement as specified at nominal flexural resistance (ksi). As specified in Article 5.6.2.1, if $c / d_s < 0.003 / (0.003 + \varepsilon_{cl})$, then f_y may used in lieu of exact computation of f_s. For 60 ksi reinforcement, ε_{cl} is taken as 0.002, making the ratio $0.003 / (0.003 + \varepsilon_{cl})$ equal to 0.6. Typically in design, f_s is assumed to be equal to f_y, then the assumption is checked.

By definition, the value of ϕ at the point of balanced strain according to Equation 5.5.4.2-2 is equal to 0.75. The point of balanced strain is the location where the tensile reinforcement yield simultaneously with the crushing of the concrete at the extreme compression fiber. The moment and axial force that comprise the factored balance point are $0.75M_b$ and $0.75P_b$.

Check Control of Cracking *(5.6.7)*

Article 12.11.3 states that the service limit state shall be calculated according to Article 5.6.7. In the commentary C12.11.3, an equation is given for the determination of the service load stresses taking into account the axial load in the member. Except for very deep fills, the effect of axial load on the service stresses is minimal and the use of the equation is not warranted.

The spacing of reinforcement, s (in.), in the layer closest to the tension face shall satisfy the following:

$$s \le \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c$$ (Eq. 5.6.7-1)

Where:

β_s = $1 + \frac{d_c}{0.7(h - d_c)}$ (Eq. 5.6.7-2)

d_c = thickness of concrete cover from extreme tension fiber to center of the flexural reinforcement located closest thereto (in.)

h = slab depth (in.)

f_{ss} = stress in mild steel tension reinforcement at service load condition, not to exceed $0.6f_y$

= $\frac{M_{SERVICE\ I}}{A_s j d_s}$ (ksi)

j = $1 - \frac{k}{3}$

k = $\sqrt{(\rho n)^2 + 2\rho n} - \rho n$

ρ = $\frac{A_s}{bd_s}$

n = $\frac{E_s}{E_c}$

E_s = 29000 ksi (6.4.1)

E_c = $120000K_1 w_c^2 f'_c{}^{0.33}$ (Eq. 5.4.2.4-1)

d_s = distance from extreme compression fiber to centroid of tensile reinforcement (in.)

K_1 = 1.0 for normal-weight concrete

w_c = 0.145 kcf (Table 3.5.1-1)

f'_c = concrete compressive strength (ksi)

γ_e = 0.75 for Class 2 Exposure. C5.6.7 defines Class 2 Exposure as decks and any substructure units exposed to water.

Check Minimum Reinforcement *(5.6.3.3)*

Article 12.11.4.3.1 states the minimum reinforcement shall be in accordance with Article 5.6.3.3.

The minimum reinforcement requirements state:

$$M_r = \phi M_n > \min(M_{cr}, 1.33M_{STRENGTH\ I})$$

Where:

$M_{cr} = \gamma_3 \gamma_1 S f_r$ (k-in.) (Eq. 5.6.3.3-1)

$S = \frac{1}{6} bh^2$ (in.^{3})

$f_r = 0.24\sqrt{f'_c}$ (ksi) (5.4.2.6)

γ_3 = 0.75 for A706, Grade 60 reinforcement

γ_1 = 1.6 for non-segmentally constructed bridges

Design Distribution and Shrinkage and Temperature Reinforcement (5.12.2.1, 5.10.6)

Distribution reinforcement is reinforcement that runs perpendicularly or obliquely to the main reinforcement. The intent of distribution reinforcement is to allow the load to disperse over a larger area than the tire patch. It is only required for culvert top slabs when the amount of fill is two feet or less; with fills greater than two feet it can be assumed that the fill itself distributes the load.

When required, this reinforcement is not designed, but rather is specified as a percentage of the main bottom reinforcement area. For top slabs, the percentage is found using the following equations:

$$\frac{100}{\sqrt{L}} \leq 50\%$$ (Eq. 5.12.2.1-1)

Where L is the span length in feet.

The Shrinkage and Temperature requirements are stated in Article 5.10.6. The required area of temperature and shrinkage reinforcement, A_s, in square inches per foot width, shall be found using the following equations:

$$A_s \geq \frac{1.30bh}{2(b+h)f_y} \quad \text{(Eq. 5.10.6-1)}$$

$$0.11 \leq A_s \leq 0.60 \quad \text{(Eq. 5.10.6-2)}$$

Where h is the least thickness of the component (in.), and b is the width of the component (in). Each slab and sidewall is calculated separately, as opposed to calculating one value for the entire culvert. Because of this, b is typically taken as the culvert width for top and bottom slabs, and culvert height for sidewalls.

Spacing for reinforcement designed for Distribution or Shrinkage and Temperature shall not exceed 12 inches center-to-center.

Check Reinforcement Spacing Requirements

The following reinforcement spacing requirements shall be observed. Some of these requirements are derived from AASHTO specifications, while others are IDOT-specific.

- The minimum spacing for primary flexural reinforcement is 5 inches.
- The maximum spacing for primary flexural reinforcement is 12 inches. Note that smaller bars with tighter spacing will provide better serviceability than larger bars with larger spacing. Reinforcement spacings of 9 inches or less are preferred.
- The minimum clear distance between adjacent lapped bars shall not be less than 4 inches.
- The need for multiple layers of bars as noted in Article 5.10.3.1.3 is generally limited to culverts with fill heights greater than about 25 feet. At this point, the theoretical bar spacing to provide approximately $0.375\rho_{bal}$ with #11 bars or smaller may be less than 5 inches. The designer has the option to either use multilayer bars or to use a thicker slab

with a smaller steel ratio. If multiple layers of bars are used, the requirements of Article 5.10.3.1.3 shall be followed.

- Bundled bars are not allowed in concrete box culverts.

Worked Example 1 – Moment Magnifier (5.6.4.3, 4.5.3.2.2)

Determine the moment magnification and the design moment for a 6 foot tall culvert sidewall. The wall is 6 inches thick and reinforced with #4 bars at 5½" centers. The maximum factored Strength I moment is 5.4 k-ft. The maximum factored Strength I permanent dead load (DC + EV) moment is 1.5 k-ft. The axial force is 13.6 kips.

f_y	=	60 ksi
f'_c	=	3.5 ksi
d_b	=	0.5 in.
Clear Cover	=	2 in.

Check Slenderness *(5.6.4.3)*

Use the provisions of Article 5.6.4.3 to determine if slenderness effects must be considered. The member is braced against sidesway. Slenderness may be neglected if $\frac{Kl_u}{r} < 34-12\frac{M_1}{M_2}$. Given that the connection of the top slab to the sidewall is considered pinned, both M_1 and M_2 are equal to zero and $\frac{Kl_u}{r} < 34$ must be satisfied to neglect slenderness effects.

Therefore check slenderness using:

$$\frac{Kl_u}{r} < 34 \qquad (5.6.4.3)$$

$K = 1.0$ (4.6.2.5)

$l_u = 72$ in.

$$r = \frac{t\sqrt{3}}{6} = \frac{(6\text{ in.})\sqrt{3}}{6} = 1.73\text{ in.}$$

$$\frac{Kl_u}{r} = \frac{(1.0)(72\text{ in.})}{1.73\text{ in.}} = 41.6 > 34 \therefore \text{slenderness effects must be checked}$$

Slenderness must be considered. Since $\frac{Kl_u}{r}$<100, use provisions of Article 4.5.3.2.2b. The maximum factored axial force is used to calculate the moment magnifier which will produce the greatest magnifier to be used to calculate M_c. The magnifier calculated for the condition of maximum axial force may be conservatively applied to the forces from any loading condition.

Calculate Moment Magnifier *(4.5.3.3.2)*

Because there is no sidesway in the box, Equation 4.5.3.2.2b-1 becomes:

$M_c = \delta_b M_{2b}$ (Eq. 4.5.3.2.2b-1)

Where:

M_{2b} = Strength I factored maximum moment

$$\delta_b = \frac{C_m}{1-\frac{P_u}{\phi_K P_e}} \geq 1.0$$ (Eq. 4.5.3.2.2b-3)

In which:

C_m = 1.0

P_u = 13.6 k

$$P_e = \frac{\pi^2 EI}{(K\ell_u)^2}$$ (Eq. 4.5.3.2.2b-5)

$$EI = \frac{E_c I_g / 2.5}{1+\beta_d}$$ (Eq. 5.6.4.3-2)

$E_c = 120000 K_1 w_c^2 f'_c{}^{0.33}$ (Eq. 5.4.2.4-1)

K_1 = 1.0 (5.4.2.4)

w_c = 0.145 kcf (Table 3.5.1-1)

f'_c = 3.5 ksi

$E_c = 120000(1.0)(0.145\text{ kcf})^2(3.5\text{ ksi})^{0.33}$

= 3815 ksi

$$I_g = \frac{bh^3}{12}$$

b = 12 in. strip width

h = 6 in. wall thickness

$$I_g = \frac{(12\ \text{in.})(6\ \text{in.})^3}{12}$$

$$= 216\ \text{in.}^4$$

β_d = ratio of maximum factored permanent load moment to maximum factored total load moment

$$= \frac{1.5\ \text{k}-\text{ft.}}{5.4\ \text{k}-\text{ft.}}$$

$$= 0.28$$

$$EI = \frac{(3815\ \text{ksi})(216\ \text{in.}^4)/2.5}{1+0.28}$$

$$= 257513\ \text{k-in.}^2$$

$$P_e = \frac{\pi^2(257513\ \text{k}-\text{in.}^2)}{((1.0)(72\ \text{in.}))^2}$$

$$= 490.3\ \text{k}$$

ϕ_K = 0.75 for concrete members (4.5.3.2.2)

$$\delta_b = \frac{1.0}{1-\frac{13.6\ \text{k}}{0.75(490.3\ \text{k})}}$$

$$= 1.04$$

The maximum moment is then 1.04(5.4 k-ft.) = 5.62 k-ft.

Worked Example 2 – Construct Axial-Moment Interaction Diagram for Wall (5.6.3.2)

Construct the axial-moment interaction diagram for a 10 in. thick wall with #6 bars at 6 in. centers. The bar cover is 2 in.

f_y = 60 ksi

f'_c = 3.5 ksi

Determine Point of Pure Flexure ($\phi P_n = 0$)

<u>Calculate Factored Resistance at Point of Pure Flexure</u>

$$M_r = \phi M_n = \phi\left[A_s f_s\left(d_s - \frac{a}{2}\right)\right] \geq M_{STRENGTH1}$$ (Eqs. 5.6.3.2.1-1 & 5.6.3.2.2-1)

Where:

ϕ = Assumed to be 0.9, then checked using the procedure found in Article 5.5.4.2.

a = $c\beta_1$

c = $\dfrac{A_s f_s}{\alpha_1 \beta_1 f'_c\, b}$ (in.) (Eq. 5.6.3.1.1-4)

A_s = 0.88 in.^{2}

b = 12 in.

d_s = 10 in. – 2 in. – 0.5(0.75 in.)

= 7.625 in.

f_s = assumed to be 60 ksi, then checked (5.6.2.1)

f'_c = 3.5 ksi

α_1 = 0.85 for concrete with strength less than 10 ksi (5.6.2.2)

β_1 = 0.85 for concrete with strength less than 4 ksi (5.6.2.2)

$$\phi M_n = 0.9\left[\left(0.88\text{ in.}^2\right)\left(60\text{ ksi}\right)\left(7.625\text{ in.} - \frac{1}{2}\frac{(0.88\text{ in.})(60\text{ ksi})}{0.85(3.5\text{ ksi})(12\text{ in.})}\right)\right]$$

= 327.2 k-in.

<u>Check ϕ = 0.9</u>

$$\varepsilon_t = \frac{0.003(d_t - c)}{c}$$ (C5.6.2.1)

Where:

d_t = d_s for members with single row of reinforcement (5.6.2.1)

= 7.625 in.

c = $\dfrac{A_s f_s}{\alpha_1 \beta_1 f'_c\, b}$ (Eq. 5.6.3.1.1-4)

$$= \frac{(0.88 \text{ in.}^2)(60 \text{ ksi})}{(0.85)(0.85)(3.5 \text{ ksi})(12 \text{ in.})}$$

$$= 1.74 \text{ in.}$$

$$= \frac{0.003\,(7.625 \text{ in.} - 1.74 \text{ in.})}{1.74 \text{ in.}}$$

= 0.01 > 0.005, therefore assumption that $\phi = 0.9$ is valid

<u>Check $f_s = 60$ ksi</u>

$\frac{c}{d} = \frac{1.73 \text{ in.}}{7.625 \text{ in.}}$ = 0.23 < 0.6, therefore assumption that $f_s = f_y = 60$ ksi is valid

Point of pure flexure occurs at (327.2 k-in., 0 k)

Determine Balance Point (0.75Mb, 0.75Pb)

$$M_b = 0.85f'_c\,\beta_1 x_b b\left(\frac{h}{2} - \frac{\beta_1 x_b}{2}\right) + A_s f_s\left(d_s - \frac{h}{2}\right)$$

$$P_b = 0.85f'_c\beta_1 x_b b - A_s f_s$$

Where:

f'_c = 3.5 ksi

β_1 = 0.85 for concrete less than 4 ksi (5.6.2.2)

$$x_b = \frac{87d_s}{f_y + 87}$$

d_s = 10 in. – 2 in. – 0.5(0.75 in.)

= 7.625 in.

f_y = 60 ksi

$$x_b = \frac{87(7.625 \text{ in.})}{60 + 87}$$

= 4.51 in.

b = 12 in.

h = 10 in.

A_s = 0.88 in.^{2}

f_s = 60 ksi (for verification of this, see above)

$$M_b = 0.85(3.5\ \text{ksi})(0.85)(4.51\ \text{in.})(12\ \text{in.})\left(\frac{10\ \text{in.}}{2} - \frac{0.85(4.51\ \text{in.})}{2}\right)$$

$$+ (0.88\ \text{in.}^2)(60\ \text{ksi})\left(7.625\ \text{in.} - \frac{10\ \text{in.}}{2}\right)$$

= 560.7 k-in.

P_b = 0.85(3.5 ksi)(0.85)(4.51 in.)(12 in.) – (0.88 in.^{2})(60 ksi)

= 84.1 k

ϕM_b = 0.75(560.7 k-in.) = 420.5 k-in.

ϕP_b = 0.75(84.1 k) = 63.1 k

Worked Example 3 – Minimum Reinforcement (5.6.3.3)

Calculate the cracking moment for an 11 in. thick wall. f'_c = 3.5 ksi, $M_{STRENGTH\ I}$ = 90 k – in., ϕM_n = 125 k – in.

$M_r = \phi M_n > \min(M_{cr}, 1.33 M_{STRENGTH\ I})$

Where:

$M_{cr} = \gamma_3 \gamma_1 S f_r$ (Eq. 5.6.3.3-1)

γ_3 = 0.75 for A706, Grade 60 reinforcement

γ_1 = 1.6 for non-segmentally constructed bridges

$S = \frac{1}{6} bh^2$

b = 12 in. strip width

h = 11 in. wall thickness

$S = \frac{1}{6}(12\ \text{in.})(11\ \text{in.})^2$

= 242 in.^{3}

$f_r = 0.24\sqrt{f'_c}$ (5.4.2.6)

$= \quad 0.24\sqrt{3.5\,\text{ksi}}$

$= \quad 0.45\text{ ksi}$

$M_{cr} = \quad 0.75(1.6)(242\text{ in.}^3)(0.45\text{ ksi})$

$= \quad 131\text{ k-in.}$

$1.33M_{STRENGTH\ I} \quad = \quad 1.33(90\text{ k-in.})$

$= \quad 119.7\text{ k-in.}$

$\text{Min}(M_{cr}, 1.33M_{STRENGTH\ I}) = 119.7\text{ k-in.}$

$\phi M_n = 125\text{ k-in.} > 119.7\text{ k-in.}$ OK

Worked Example 4 – Control of Cracking (5.6.7)

Verify the reinforcement bar spacing for an 11 in. thick wall with reinforcement of #7 at 7 in. centers. The reinforcement is A706 Grade 60. The bar cover is 2 in. The maximum Service I moment is 14.7 k-ft./ft. The minimum Service I axial force is 4.1 k/ft.

$f_y = 60\text{ ksi}$ $d_b = 0.875\text{ in.}$

$f'_c = 3.5\text{ ksi}$ $A_{bar} = 0.60\text{ in.}^2$

Verify Spacing Limitations

Use Article 5.6.7 and conservatively neglect the effect of axial force on the calculation of the bar stresses as discussed in C12.11.3. If the section does not satisfy the requirements with this conservative assumption, the provisions of C12.11.3 will be used.

$$s \le \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c \qquad \text{(Eq. 5.6.7-1)}$$

Where:

$\gamma_e \quad = \quad 0.75$ for Class 2 Exposure conditions

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} \qquad \text{(Eq. 5.6.7-2)}$$

In which:

d_c = thickness of concrete cover from extreme tension fiber to center of the flexural reinforcement located closest thereto (in.)

= 2 in. clear + 0.5(0.875 in.)

= 2.4375 in.

h = slab depth (in.)

= 11 in.

$$\beta_s = 1 + \frac{2.4375 \text{ in.}}{0.7(11 \text{ in.} - 2.4375 \text{ in.})}$$

= 1.41

f_{ss} = stress in mild steel tension reinforcement at service load condition, not to exceed $0.6f_y$

$$= \frac{M_{SERVICE\ I}}{A_s j d_s} \text{ (ksi)}$$

In which:

$M_{SERVICE\ I}$ = 14.7 k-ft. / ft. (given)

$$j = 1 - \frac{k}{3}$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$\rho = \frac{A_s}{bd_s}$$

A_s = 1.03 in.^{2}

b = 12 in.

d_s = 11 in. – 2 in. clear – 0.5(0.875 in.)

= 8.56 in.

$$\rho = \frac{1.03 \text{ in.}^2}{(12 \text{ in.})(8.56 \text{ in.})}$$

= 0.010

$$n = \frac{E_s}{E_c}$$

E_s = 29000 ksi (6.4.1)

E_c = $120000K_1w_c^2f'_c{}^{0.33}$ (Eq. 5.4.2.4-1)

K_1 = 1.0 for normal-weight concrete

w_c = 0.145 kcf (Table 3.5.1-1)

f'_c = 3.5 ksi

E_c = 120000(1.0)(0.145 kcf)^2(3.5 ksi)$^{0.33}$

= 3815 ksi

$$n = \frac{29000\ \text{ksi}}{3815\ \text{ksi}}$$

= 7.27

$$k = \sqrt{[(0.010)(7.27)]^2 + 2(0.010)(7.27)} - (0.010)(7.27) = 0.315$$

$$j = 1 - \frac{0.315}{3} = 0.895$$

$$f_{ss} = \frac{14.7\ \text{k} - \text{ft.}\left(\frac{12\ \text{in.}}{\text{ft.}}\right)}{(1.03\ \text{in.}^2)(0.895)(8.56\ \text{in.})}$$

= 22.35 ksi < 0.6(60 ksi) = 36 ksi O.K.

$$\frac{700\gamma_e}{\beta_s f_s} - 2d_c = \frac{700(0.75)}{(1.41)(22.35\ \text{ksi})} - 2(2.4375\ \text{in.}) = 11.8\ \text{in.}$$

s = 7 in. < 11.8 in. O.K.

∴#7 bars @ 7 in. center-to-center spacing is adequate to control cracking.

Alternative Solution Taking the Axial Load into Account

Use Eq. C12.11.3-1 to calculate the stress in the bar. The axial force, N_s, has been calculated to be 4.1 kips per foot. Recalculate the maximum bar spacing using the minimum Service I axial force.

$$f_s = \frac{M_s + N_s\left(d - \frac{h}{2}\right)}{A_s jid} \qquad \text{(Eq. C12.11.3-1)}$$

Where:

$$e = \frac{M_s}{N_s} + d - \frac{h}{2}$$

M_s = 176 k – in. / ft.

N_s = 4.1 k / ft.

d = 8.56 in.

h = 11 in.

$$e = \frac{176\ \text{k} - \text{in./ft.}}{4.1\ \text{k/ft.}} + 8.56\ \text{in.} - \frac{11\ \text{in.}}{2}$$

$$= 45.98\ \text{in.}$$

$$j = 0.74 + \frac{0.1e}{d} \leq 0.9$$

$$= 0.74 + \frac{0.1(45.98\ \text{in.})}{8.56\ \text{in.}}$$

$$= 1.28 > 0.9,\ \therefore\ j = 0.9$$

$$i = \frac{1}{1 - \frac{(0.9)(8.56\ \text{in.})}{45.98\ \text{in.}}}$$

$$= 1.20$$

$$f_s = \frac{176\ \text{k} - \text{in./ft.} + 4.1\ \text{k/ft.}\left(8.56\ \text{in.} - \frac{11\ \text{in.}}{2}\right)}{(1.03\ \text{in.}^2)(0.9)(1.20)(8.56\ \text{in.})}$$

$$= 19.80\ \text{ksi}$$

$$\frac{700\gamma_e}{\beta_s f_s} - 2d_c = \frac{700\,(0.75)}{(1.41)(19.80\ \text{ksi})} - 2(2.4375\ \text{in.}) = 13.9\ \text{in.}$$

s = 7 in. < 13.9 in. O.K.

Spacing requirements are satisfied.